

CHAPTER 9

SEEPAGE LOSSES FROM SUBIRRIGATION AND WATER TABLE CONTROL SYSTEMS

Introduction

One of the most important components of a subirrigation system is the development of a water supply with adequate capacity to meet plant use requirements, plus replenish water lost from the system by seepage. When the water table is raised during subirrigation, the hydraulic head in the field is higher than that in surrounding areas and water is lost from the system by lateral seepage. The rate of deep seepage or vertical water movement from the soil profile may also be increased. The magnitude of seepage losses depend on the hydraulic conductivity of the soil and depth to restricting layers. It also depends on boundary conditions such as the elevation of the controlled water table in relation to surrounding water table depths and the distance to drains or canals that are not controlled. Methods for characterizing seepage losses from subirrigated fields are presented in the following sections. The methods used are similar in concept to those described by Hall (1976) for computing reservoir water losses, as affected by ground water mounds. However, water tables are usually high for subirrigation systems and seepage losses can be computed by considering flow in one or two dimensions, whereas, the reservoir seepage problem is normally a two or three dimensional problem.

Seepage Losses to Nearby Drains or Canals

Methods for quantifying steady seepage losses in the lateral direction can be developed by considering the case shown in Figure 9-1. Using the Dupuit-Forchheimer (D-F) assumptions, the seepage rate may be expressed as,

$$q = - Kh \frac{dh}{dx} \quad (9-1)$$

Where q is the seepage rate per unit length of the drainage ditch (or per unit thickness into the paper ($\text{cm}^3/\text{cm hr}$ or $\text{ft}^3/\text{ft hr}$)). K is the effective lateral hydraulic conductivity (cm/hr). h is the water table elevation above the impermeable layer (cm or ft), which is a function of the horizontal position, x . If evapotranspiration from the surface is assumed negligible, q is constant for all x and Equation (9-1) can be solved, subject to the boundary conditions,

$$h = h_1 \text{ at } x = 0 \quad (9-2)$$

$$h = h_2 \text{ at } x = s \quad (9-3)$$

The solution for h may be written as,

$$h^2 = - \frac{h_1^2 - h_2^2}{s} x + h_1^2 \quad (9-4)$$

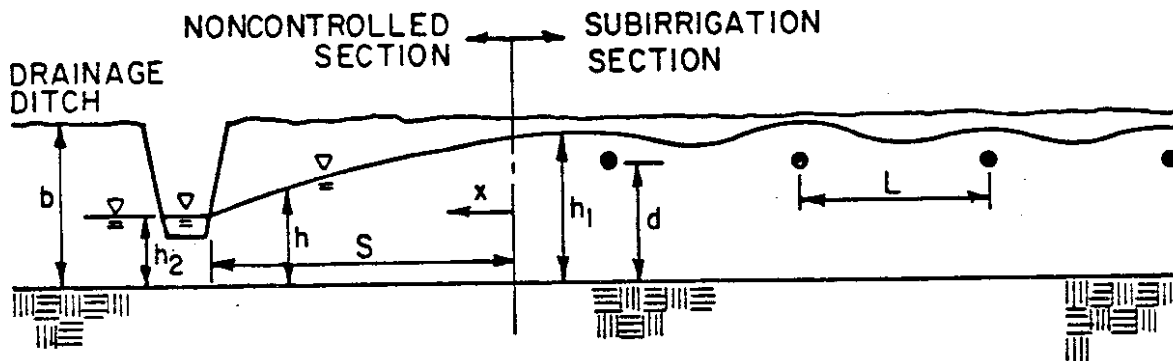


Figure 9-1. Water table profile for seepage from a subirrigated field to a drainage ditch.

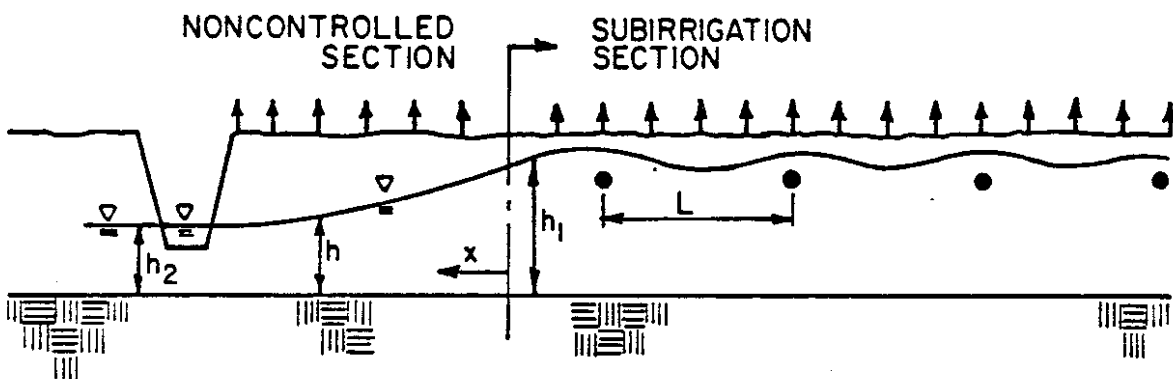


Figure 9-2. Water table profile for seepage from a subirrigated field to a drainage ditch. ET losses are considered.

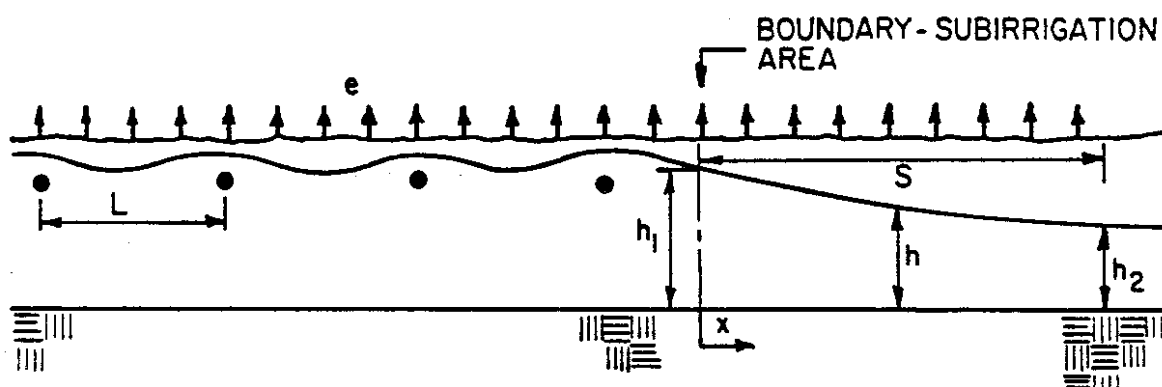


Figure 9-3. Seepage from a subirrigated field to an adjacent nonirrigated field which has water table drawdown due to ET.

Differentiating Equation (9-4) and substituting back into (9-1) gives,

$$q = \frac{K}{2S} (h_1^2 - h_2^2) \quad (9-5)$$

Then, if the length of the field (into the paper) is ℓ , the seepage loss from that side of the field may be calculated as,

$$Q = q \ell = \frac{K\ell}{2S} (h_1^2 - h_2^2) \quad (9-6)$$

Vertical water losses due to ET along the field boundaries increase the hydraulic gradients in the horizontal direction and, thus, seepage losses (Figure 9-2). In this case, the flux, q , may still be expressed by Equation (9-1), but rather than the flux being constant we may write, according to Harr (1962),

$$\frac{dq}{dx} = -e \quad (9-7)$$

Where e is the ET rate.

Then, substituting Equation 9-1 for q ,

$$\frac{d}{dx} \left[h \frac{dh}{dx} \right] = \frac{e}{K} \quad (9-8)$$

Solving (9-8), subject to boundary conditions (9-2) and (9-3) gives,

$$h^2 = \frac{e}{K} x^2 + \frac{(h_2^2 - h_1^2 - \frac{e}{K} S^2)}{S} x + h_1^2 \quad (9-9)$$

Again, differentiating and evaluating dh/dx at $x = 0$ and substituting into (9-1) yields,

$$q = \frac{K (h_1^2 - h_2^2) + e S^2}{2S} \quad (9-10)$$

Notice that for no ET ($e = 0$), Equations (9-9) and (9-10) reduce to (9-4), and (9-5), respectively, as they should.

Seepage Losses to Adjacent Undrained Lands

Subirrigation systems are often located next to forest or cropland that is not drained. However, seepage losses may still occur along these boundaries because of low water tables in the undrained areas. Why would water tables be low in surrounding areas if they are not drained? Remember that subirrigation is used during dry period so water tables would be drawn down due to ET. Such a situation is shown schematically in Figure 9-3. The problem here, as opposed to the cases above is that neither h_2 nor S is

known. The relationship between the rate of steady upward water movement and water table depth was discussed in an earlier section (pages 5-13 to 5-23). For purposes of this problem, it is assumed that water will not move to the surface (or to the root zone) at a rate sufficient to support an ET rate of e for water table elevations less than h_2 . Then, from principles of conservation of mass, we may write for any point, x ,

$$q(x) = (S-x) e \quad (9-11)$$

Where $q(x)$ is the flowrate per unit length of the field (into the paper) expressed as a function of x , e is the steady ET rate, S is the limiting distance where $h = h_2$, the limiting water table elevation that will allow upward water movement to the surface at rate e .

Substituting Equation 9-1 for q gives,

$$- Kh \frac{dh}{dx} = (S - x) e \quad (9-12)$$

Separating variables and integrating subject to the condition $h = h_1$ at $x = 0$, yields the following expression for h ,

$$h^2 = \frac{ex^2}{K} - \frac{2 S ex}{K} + h_1^2 \quad (9-13)$$

Then, S can be determined by substituting $h = h_2$ at $x = S$, which after simplifying results in,

$$S = \frac{\sqrt{(h_1^2 - h_2^2) K}}{e} \quad (9-14)$$

Then, the seepage loss per unit length of the field may be evaluated from Equation (9-11) at $x = 0$ as,

$$q = \sqrt{\frac{(h_1^2 - h_2^2) K}{e}} e \quad (9-15)$$

or

$$q = \sqrt{(h_1^2 - h_2^2) K} e \quad (9-16)$$

Normally, seepage losses to surrounding undrained areas would be highest during peak consumptive use periods. The value of h_1 would depend on the water level held in the subirrigation system. The value of h_2 would depend on the soil profile and could be chosen from relationships for maximum upward flux versus water table depth (Figure 9-6). To be on the safe side h_2 should be chosen so that the depth of the water table is at least 1.0 m at $x = S$.

Vertical or Deep Seepage

Subirrigation and water table control systems are usually located on soils with tight underlying layers and/or high natural water tables so that

vertical losses are not excessive. When evaluating a potential site for a subirrigation system, vertical seepage losses under a raised water table condition should be estimated even though a natural high water table is known to exist. These losses should be added to lateral seepage estimates to determine the water supply capacity needed in addition to that required to meet ET demands.

Deep seepage can be estimated for soils with restricting layers at a relatively shallow depth by a straight-forward application of Darcy's law. Referring to Figure 9-4, the vertical seepage flux may be estimated as,

$$q_v = K_v \frac{h_1 - h_2}{D} \quad (9-17)$$

Where q_v is the flux (m/day), K_v is the effective vertical hydraulic conductivity of the restricting layer, h_1 is the average distance from the bottom of the restricting layer to the water table, h_2 is the hydraulic head in the ground water aquifer referenced to the bottom of the restricting layer, and D is the thickness of the restricting layer.

The hydraulic head in the ground water aquifer may be estimated from the water level in wells in the vicinity. It may be necessary to install piezometers to the depth of the ground water aquifer in order to accurately determine the hydraulic head in the aquifer. Methods for installing the piezometers are discussed in Section 16 of NEH (pages 81-87). The thickness and hydraulic conductivity of the restricting layer may be determined from deep borings in the field. Data from such borings should be logged in accordance with the procedures given in Section 16 of NEH (pages 63-70). The vertical hydraulic conductivity, K_v , of restricting layers can be determined from in-field pumping tests using the piezometer method (see Bouwer and Jackson, 1974). Laboratory tests on undisturbed cores can also be used to determine K_v ; however, field tests are preferred, when possible.

The restricting strata is often composed of several layers of different conductivities and thicknesses rather than a single layer. In this case, K_v in Equation (9-17), is replaced by the effective vertical hydraulic conductivity K_{ve} . The effective conductivity can be calculated for flow perpendicular to a series of layers (Harr, 1922) as,

$$K_{ve} = \frac{D}{\frac{D_1}{K_{v1}} + \frac{D_2}{K_{v2}} + \frac{D_3}{K_{v3}} + \dots} \quad (9-18)$$

Where $D_1, D_2, D_3, \dots$ are the thicknesses, and $K_{v1}, K_{v2}, K_{v3}, \dots$ are the vertical hydraulic conductivities of the individual layers.

Examples

An example layout of a subirrigation system is shown in Figure 9-5. Drains are placed 20 m apart and the water level directly above the drains is held to within 50 cm of the surface during the growing season. Seepage losses occur along all four boundaries of the field. The effective lateral hydraulic conductivity is 2.0 m/day for the field and surrounding areas, except for the compacted roadway south of the field where $K = 0.5$ m/day.

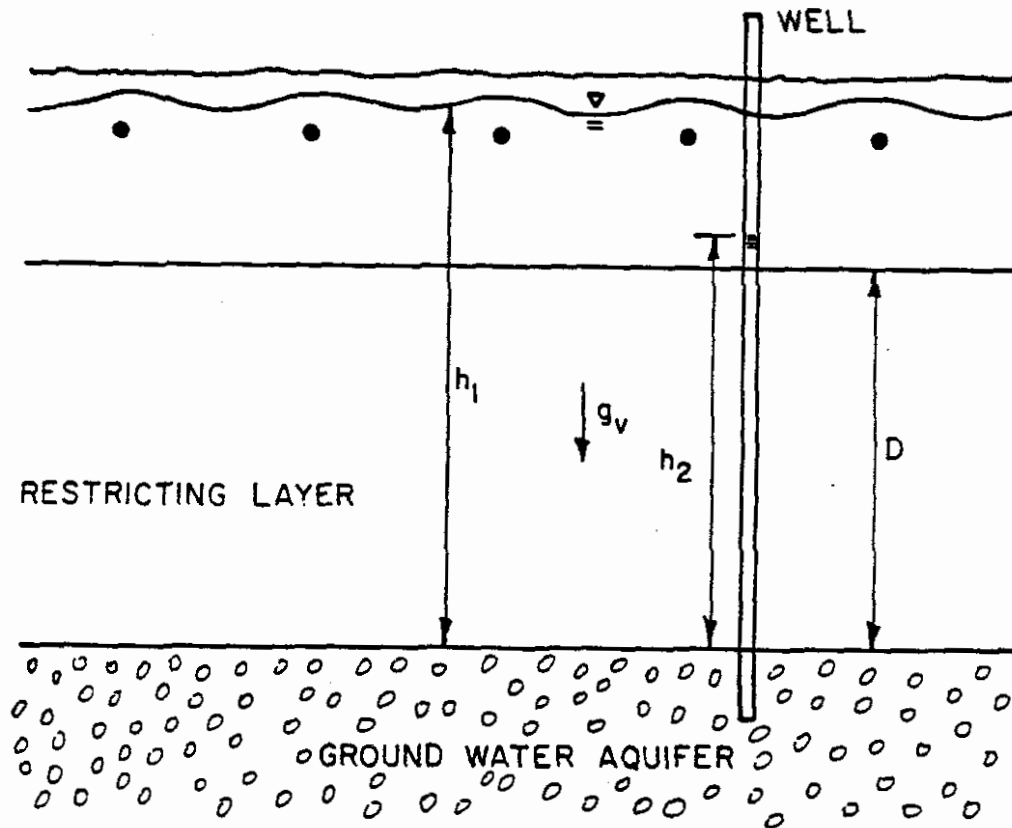


Figure 9-4. Vertical seepage to a ground water aquifer during subirrigation.

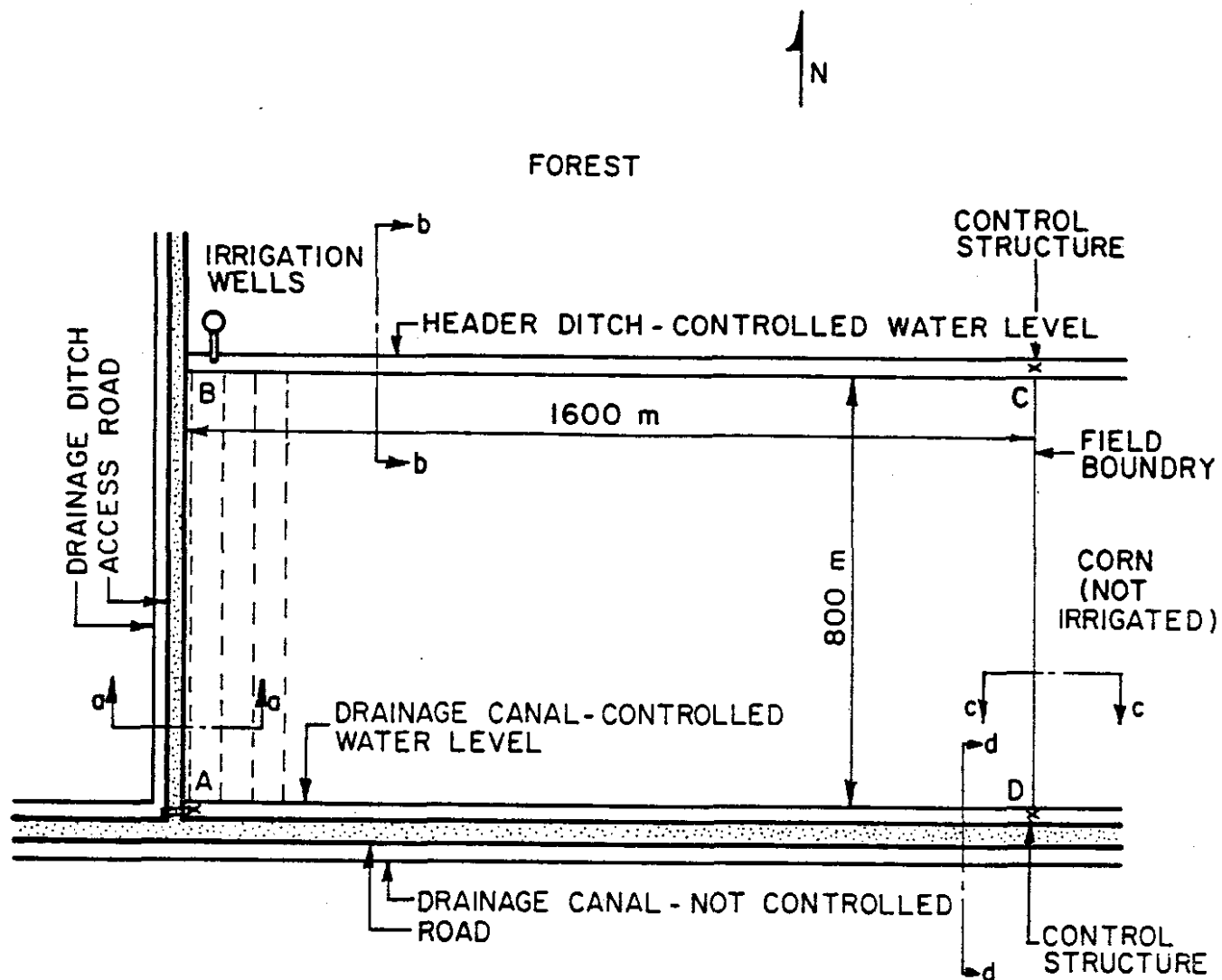


Figure 9-5. Schematic of a 128 ha (307 acre) subirrigation system showing boundary conditions for calculating lateral seepage losses.

Boundary A-B

Along Boundary A-B, water moves from the field under a 5 m wide uncompacted field access road to a drainage ditch on the other side (Figure 9-6a). A drain tube is located immediately adjacent to the road in order to maintain good water table control right up to the field boundary. The seepage rate under the road can be calculated using Equation 9-5 as,

$$q_{A-B} = \frac{2.0 \text{ m/day}}{2 \times 5 \text{ m}} (1.5^2 - 0.6^2) \text{ m}^2 = 0.378 \frac{\text{m}^3}{\text{day m}}$$

$$Q_{A-B} = q \ell = 0.378 \text{ m}^3/\text{m day} \times 800 \text{ m} = 302 \text{ m}^3/\text{day}$$

Converted to more familiar units, the seepage rate may be written as,

$$Q_{A-B} = 302 \text{ m}^3/\text{day} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times (3.28 \frac{\text{ft}}{\text{m}})^3 \times \frac{7.5 \text{ gal}}{\text{ft}^3}$$

$$Q_{A-B} = 55 \text{ gal/min}$$

This rather high seepage loss can be reduced by moving the first lateral away from the edge of the field, say by one-half of the drain spacing (Figure 9-6b). Then, substituting $S = 10 + 5 = 15 \text{ m}$ in Equation 9-6 gives,

$$Q_{A-B} = \frac{2.0 \text{ m/day} \times 800 \text{ m}}{2 \times 15 \text{ m}} (1.5^2 - 0.6^2) = 100 \text{ m}^3/\text{day}$$

or

$$Q_{A-B} = 18 \text{ gal/min}$$

This would be the seepage rate when $ET = e = 0$. Seepage losses are most critical during periods of high consumptive use (high ET by crop) because it is at this period that the highest supply rate will be required. The seepage rate for a design ET value of $e = 0.6 \text{ cm/day}$ can be calculated from Equation 9-10 as,

$$q_{A-B} = \frac{2.0 \text{ m/day} (1.5^2 - 0.6^2) \text{ m}^2 + .006 \text{ m/day} \times 15^2 \text{ m}^2}{2 \times 15 \text{ m}}$$

$$q_{A-B} = 0.171 \text{ m}^3/\text{m day}$$

$$Q_{A-B} = q \ell = 0.171 \text{ m}^3/\text{m day} \times 800 \text{ m} = 137 \text{ m}^3/\text{day}$$

or

$$Q = 25 \text{ gal/min}$$

However, it should be noted that this is the flowrate from the first lateral toward the access road and the adjacent drainage ditch. Part of the water supplies the ET demand between the lateral and the ditch and should not be counted as seepage loss. The rate of water used in the 10 m strip between the first lateral and the access road is,

$$\begin{aligned} Q_e &= 0.006 \text{ m/day} \times 10 \text{ m} \times 800 \text{ m} \\ &= 48 \text{ m}^3/\text{day} \end{aligned}$$

then

$$Q_{A-B} = 137 \text{ m}^3/\text{day} - 48 = 89 \text{ m}^3/\text{day} = 16 \text{ gal/min}$$

This includes water lost by seepage to the drainage ditch plus water lost by ET from the road surface (at an assumed rate of 0.6 cm/day) where grass, weeds, etc., are growing. Note that the same result would have been obtained by evaluating the quantity $h \, dh/dx$ from Equation (9-9) at $x = 10 \text{ m}$, rather than at $x = 0$. Then, Equation 9-10 would have been replaced by,

$$q = -e \, x + \frac{K}{2S} (h_1^2 - h_2^2 + \frac{e}{K} S^2) \quad (9-19)$$

and

$$q_{A-B} = -.006 \times 10 + \frac{2.0}{2 \times 15} (1.5^2 - .6^2 + \frac{.006}{2} \times 15^2)$$

$$q_{A-B} = 0.111 \text{ m}^3/\text{day m}$$

$$Q_{A-B} = 0.111 \times 800 = 88.8 \text{ m}^3/\text{day} = 16 \text{ gal/min}$$

which is the same as determined above.

It is interesting that seepage losses for $e = 0$ are greater than for $e = 0.6 \text{ cm/day}$. The reason for this is that ET within the field lowers the water table elevation at the field edge and thus the hydraulic gradient and seepage rates are reduced. Losses can be further reduced by moving the first lateral further away from the field boundary. This may mean sacrificing the quality of water table control near the edge of the field, but should be considered if seepage losses are excessive.

Boundary B-C

Seepage losses along the North Boundary, B-C, are in response to gradients caused by water table drawdown due to ET, as shown schematically in Figure 9-7. The relationship between maximum upward flux and water table depth (Figure 5-6) indicate that, for the Lumbee soil, an ET rate of 0.6 cm/day can be sustained with a water table depth below the root zone of 50 cm and a rate of 0.2 cm/day at a depth of 60 cm. Assuming an effective

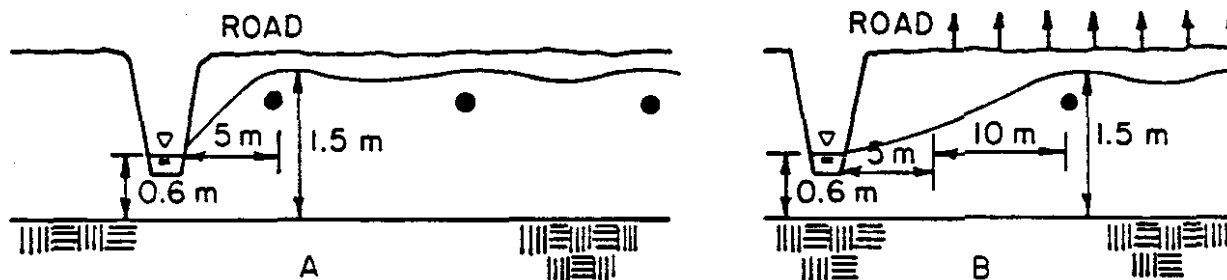


Figure 9-6. Seepage along Boundary A-B: (a) the first drain tube is located immediately adjacent to the field access road 5 m from the drainage ditch, and (b) the drain tube is located 10 m back from the road.

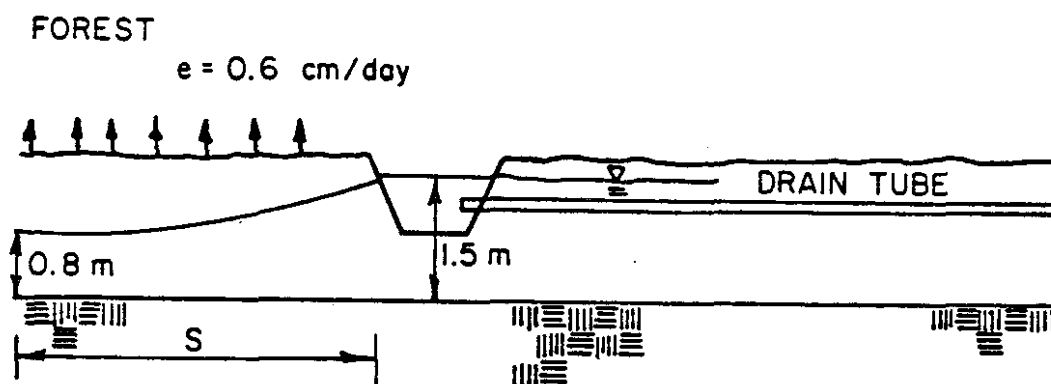


Figure 9-7. Schematic of water table position along the North Boundary (Section B-B).

rooting depth of 60 cm (2 ft) and taking a conservative estimate of 60 cm for the water table depth below the root zone, gives a total water table depth of 1.2 m and $h_2 = 2.0 - 1.2 = 0.8$ m. Then, the seepage rate can be determined from Equation 9-16 as,

$$\begin{aligned} q_{B-C} &= \sqrt{(1.5^2 - 0.8^2)} \cdot 2.0 \times 0.006 \text{ m}^3/\text{m day} \\ &= 0.139 \text{ m}^3/\text{m day} \end{aligned}$$

and

$$Q_{B-C} = 1,600 q_{B-C} = 222 \text{ m}^3/\text{day} = \underline{41 \text{ gal/min}}$$

Seepage along B-C increases with the square root of e in contrast to Boundary A-B where seepage losses decrease with increasing e . It is also interesting to note that a 25 percent increase in h_2 to 1.0 m still gives a seepage rate of 36 gal/min, a reduction of only 12 percent.

Boundary C-D

As in the previous case, seepage losses along BC are caused by a lower water table in the adjacent nonirrigated field which was drawn down by ET (Figure 9-8). By assuming an effective maximum root depth for corn of 30 cm and a water table depth below the root zone of 60 cm ($y = 0.60 + 0.30 = 0.90$ so $h_2 = 2.0 - 0.90 = 1.1$) for a steady ET rate of $e = 0.6$ cm/day, the seepage rate from the last drain tube toward the boundary C-D is (Equation 9-16),

$$q = \sqrt{(1.5^2 - 1.1^2)} \cdot 2.0 \times 0.006 = 0.112 \text{ m}^3/\text{m day}$$

However, part of this seepage supplies the ET demand for the region between the last tube and the field boundary and should not be considered as seepage loss. If the last drain tube is located 10 m from the edge of the field, the portion of the above seepage used by ET within the irrigated field is, $q_e = 0.006 \text{ m/day} \times 10 \text{ m} = 0.06 \text{ m}^3/\text{m day}$. Therefore,

$$q_{C-D} = 0.112 - 0.06 = 0.052 \text{ m}^3/\text{m day}$$

and

$$Q_{C-D} = 0.052 \text{ m}^3/\text{m day} \times 800 \text{ m} = 41 \text{ m}^3/\text{day} = \underline{7.5 \text{ gal/min}}$$

An alternative means of calculating this loss is to first determine S for which $h = h_2 = 1.1$ m from Equation (9-14).

$$S = \sqrt{(1.5^2 - 1.1^2)} \cdot 2.0 / 0.006 = 18.6 \text{ m}$$

And, then determine q_{C-D} from Equation (9-19) with $x = 10$ m,

$$q_{C-D} = .052 \text{ m}^3/\text{m day}$$

which is the same value obtained above.

Boundary A-D

Seepage under the road along Boundary A-D (Figure 9-9) can be estimated using Equation 9-6 with K for the compacted road fill of 0.5 m/day.

$$Q_{A-D} = \frac{0.5 \text{ m/day} \times 1,600 \text{ m}}{2 \times 15 \text{ m}} (1.5^2 - 0.7^2) \text{ m}^2$$

$$Q_{A-D} = 47 \text{ m}^3/\text{day} = 8.5 \text{ gal/min}$$

Deep Seepage

Deep borings and hydraulic conductivity tests using the piezometer method indicate the thickness of the restricting layer is 20 m with an effective vertical hydraulic conductivity of $K_v = 0.01 \text{ cm/hr}$. Measurements in observation wells, cased to the depth of the ground water aquifer (22 m deep), show a nearly constant hydraulic head of $h_2 = 20.5 \text{ m}$ (refer to Figure 9-4). Then, assuming an average $h_2 = 21.3 \text{ m}$, the vertical seepage rate can be calculated from Equation (9-17) as,

$$q_v = 0.01 \text{ cm/hr} \frac{21.3 \text{ m} - 20.5 \text{ m}}{20 \text{ m}}$$

$$q_v = 0.0004 \text{ cm/hr} = 0.000096 \text{ m/day}$$

Then, for the entire field with dimensions of 800 m x 1,600 m, the vertical seepage rate is,

$$Q_v = q_v A = .000096 \times 800 \times 1,600 = 123 \text{ m}^3/\text{day} = 22 \text{ gal/min}$$

Total Seepage Losses

Based on the previous calculations, the total seepage losses are:

$$Q_T = Q_{A-B} + Q_{B-C} + Q_{C-D} + Q_{A-D} + Q_v$$

$$Q_T = 89 + 222 + 41 + 47 + 123 = 522 \text{ m}^3/\text{day}$$

or

$$Q_T = 95 \text{ gal/min}$$

This amount of water will have to be supplied in addition to the irrigation water necessary to satisfy ET demand during the operation of the subsurface irrigation system. The calculations are based on a peak ET rate of 0.6 cm/day. Therefore, the capacity required to satisfy ET during periods of dry weather when the total demand must be satisfied by the subirrigation system is,

$$Q_{ET} = 0.6 \text{ cm/day} \times \frac{1 \text{ m}}{100 \text{ cm}} \times 800 \text{ m} \times 1,600 \text{ m}$$

$$Q_{ET} = 7,680 \text{ m}^3/\text{day} \text{ or } \underline{1,400 \text{ gpm}}$$

Thus, the seepage loss expressed as a percentage of the total capacity is:

$$\text{Percentage loss} = 522/8,200 \times 100 = \underline{6.4 \text{ percent}}$$

which is quite reasonable, compared to conventional methods of irrigation.