

## CHAPTER 2

### THE MODEL

#### Background

A schematic of the type of water management system considered is given in Figure 2-1. The soil is nearly flat and has an impermeable layer at a relatively shallow depth. Subsurface drainage is provided by drain tubes or parallel ditches at a distance  $d$ , above the impermeable layer and spaced a distance,  $L$ , apart. When rainfall occurs, water infiltrates at the surface and percolates through the profile raising the water table and increasing the subsurface drainage rate. If the rainfall rate is greater than the capacity of the soil to infiltrate, water begins to collect on the surface. When good surface drainage is provided so that the surface is smooth and on grade, and outlets are available, most of the surface water will be available for runoff. However, if surface drainage is poor, a certain amount of water must be stored in depressions before runoff can begin. After rainfall ceases, infiltration continues until the water stored in surface depressions is infiltrated into the soil. Thus, poor surface drainage effectively lengthens the infiltration event for a given storm permitting more water to infiltrate and a larger rise in the water table than would occur if depression storage did not exist.

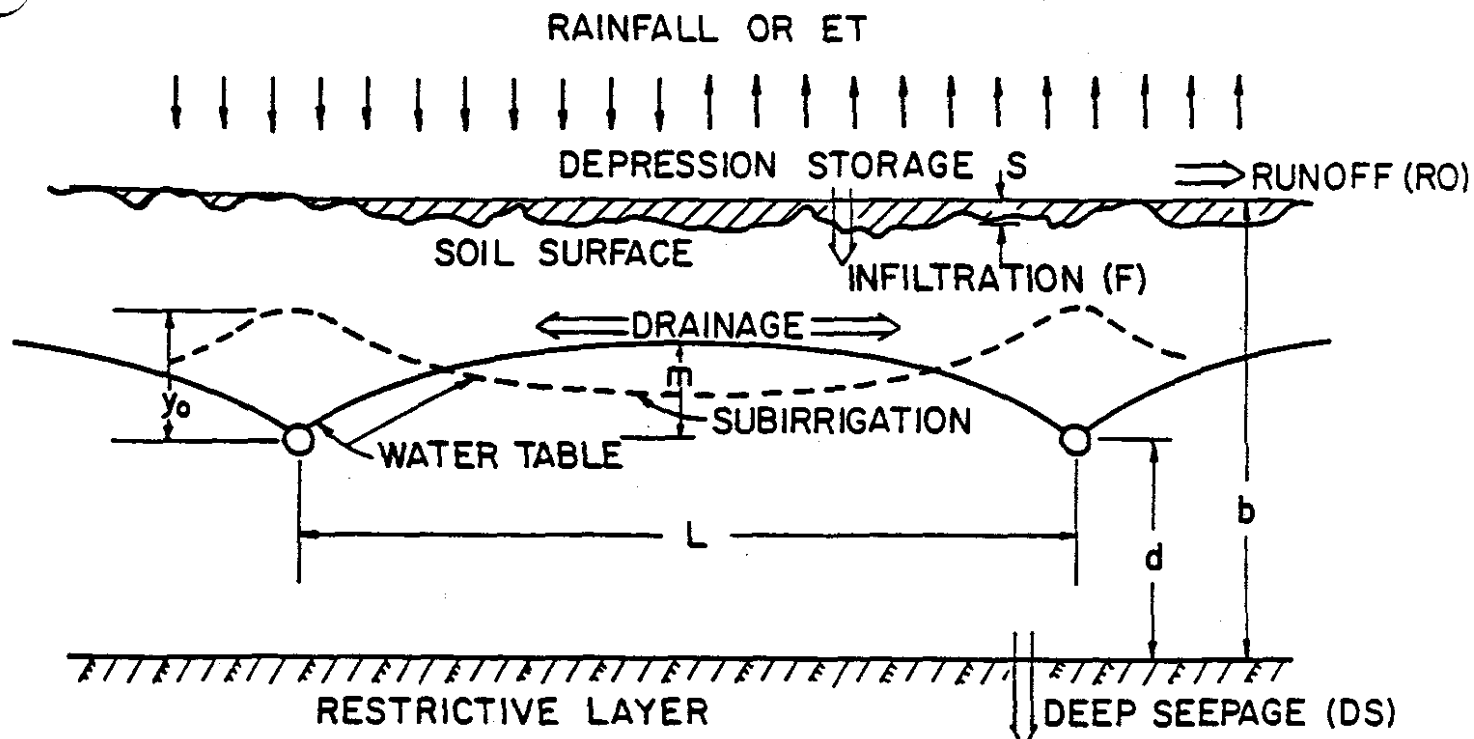


Figure 2-1. Schematic of water management system with subsurface drains that may be used for drainage or subirrigation.

The rate water is drained from the profile depends on the hydraulic conductivity of the soil, the drain depth and spacing, the effective profile depth, and the depth of water in the drains. When the water level is raised in the drains for purposes of supplying water to the root zone of the crop, the drainage rate will be reduced and water may move from the drains into the soil profile giving the shape shown by the broken curve in Figure 2-1. Studies by Skaggs (1974) showed that a high water table reduces the amount of storage available for infiltrating rainfall and may result in frequent conditions of excessive soil water if the system is not properly designed and managed. Water may also be removed from the profile by evapotranspiration (ET) and by deep seepage, both of which must be considered in the calculations if the soil water regime is to be modeled successfully.

#### Model Development

Two important criteria were adopted in the development of the computer model. First, the model must be capable of characterizing all aspects of water movement and storage in the profile so as to predict, as accurately as possible, the soil water regime and drainage rates with time. And second, the model must be developed such that the computer time necessary to simulate long-term events is not prohibitive. The movement of water in soil is a complex process; it would be an easy matter to become so involved with getting exact solutions to every possible situation that the final answer

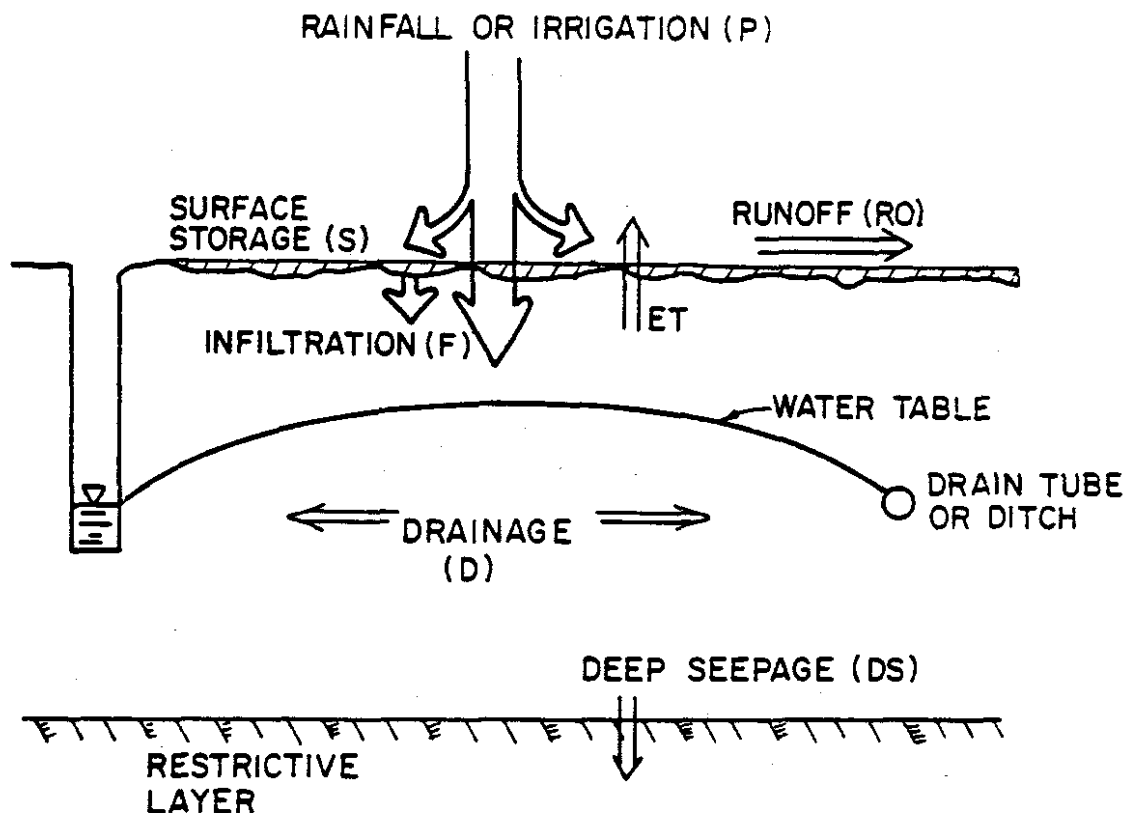


Figure 2-2. Schematic of water management system with drainage to ditches or drain tubes. Components considered in the water balance are shown on the diagram.

would never be obtained. The guiding principle in the model development was therefore to assemble the linkage between various components of the system, allowing the specifics to be incorporated as subroutines, so that they can readily be modified when better methods are developed.

The basis for the computer model is a water balance for the soil profile (Figure 2-2). The rates of infiltration, drainage, and evapotranspiration, and the distribution of soil water in the profile can be computed by obtaining numerical solutions to nonlinear differential equations (e.g., Freeze, 1971). However, these methods are impractical for our purposes because they require prohibitive amounts of computer time for long-term simulations. Instead, approximate methods were used to characterize the water movement processes. In order to insure that the approximate methods provided reliable estimates, they were compared to exact methods for a range of soils and boundary conditions. Further, the reliability of the total model was tested using field experiments.

The basic relationship in the model is a water balance for a thin section of soil of unit surface area which extends from the impermeable layer to the surface and is located midway between adjacent drains. The water balance for a time increment of  $\Delta t$  may be expressed as,

$$\Delta V_a = D + ET + DS - F \quad (2-1)$$

Where  $\Delta V_a$  is the change in the air volume (cm),  $D$  is lateral drainage (cm) from (or subirrigation into) the section,  $ET$  is evapotranspiration (cm),  $DS$  is deep seepage (cm), and  $F$  is infiltration (cm) entering the section in  $\Delta t$ .

The terms on the right-hand side of Equation 2-1 are computed in terms of the water table elevation, soil water content, soil properties, site and drainage system parameters, crop and stage of growth, and atmospheric conditions. The amount of runoff and storage on the surface is computed from a water balance at the soil surface for each time increment which may be written as,

$$P = F + \Delta S + RO \quad (2-2)$$

Where  $P$  is the precipitation (cm),  $F$  is infiltration (cm),  $\Delta S$  is the change in volume of water stored on the surface (cm), and  $RO$  is runoff (cm) during time  $\Delta t$ . The basic time increment used in Equations 2-1 and 2-2 is 1 hour. However, when rainfall does not occur and drainage and  $ET$  rates are slow such that the water table position moves slowly with time, Equation 2-1 is based on  $\Delta t$  of 1 day. When drainage is rapid but no rainfall occurs,  $\Delta t = 2$  hours is used. Conversely, time increments of 0.05 hours or less are used to compute  $F$  when rainfall rates exceed the infiltration capacity. A general Flow Chart for DRAINMOD is given in Figure 2-3. Methods used to evaluate the terms in Equations 2-1 and 2-2 and other model components are discussed in the following sections.



## Model Components

### Precipitation

Precipitation records are one of the major inputs of DRAINMOD. The accuracy of the model prediction for infiltration, runoff, and surface storage is dependent on the complete description of rainfall. Therefore, a short time increment for rainfall input data will allow better estimates for these model components than with less frequent data. A basic time increment of one hour was selected for use in the model because of the availability of hourly rainfall data. While data for shorter time increments are available for a few locations, hourly rainfall data are readily available for many locations in the United States.

Hourly rainfall records are stored in the computer based HISARS (Wiser, 1972, 1975) for several locations in North Carolina and these records are automatically accessed as inputs to the model. A data set for selected locations (at least 2 per state where possible) in the eastern USA is now being developed at North Carolina State University. These hourly rainfall and daily maximum and minimum temperature data will be available to the SCS and to other public and private agencies and will permit the use of DRAINMOD for a wide variety of climatic and geographic conditions. Hourly data for other locations in the USA can be obtained from the National Weather Service at Asheville, North Carolina.

### Infiltration

Infiltration of water at the soil surface is a complex process which has been studied intensively during the past two decades. A recent review of infiltration and methods for quantifying infiltration rates was presented by Skaggs, et al, (1979), Philip (1969), Hilel (1971), Morel-Seytoux (1973), and Hadas, et al, (1973) have also presented reviews of the infiltration processes. Infiltration is affected by soil factors such as hydraulic conductivity, initial water content, surface compaction, depth of profile, and water table depth; plant factors such as extent of cover and depth of root zone; and climatic factors such as intensity, duration, and time distribution of rainfall, temperature, and whether or not the soil is frozen.

Methods for characterizing the infiltration process have concentrated on the effects of soil factors and generally assume the soil system to be a fixed or undeformable matrix with well-defined hydraulic conductivity and soil water characteristic functions. Under these assumptions and the additional assumption that there is negligible resistance to the movement of displaced air, the Richards equation may be taken as the governing relationship for the process. For vertical water movement, the Richards equation may be written as,

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \left[ K(h) \frac{\partial h}{\partial z} \right] - \frac{\partial K(h)}{\partial z} \quad (2-3)$$

Where  $h$  is the soil water pressure head,  $z$  is the distance below the soil surface,  $t$  is time,  $K(h)$  is the hydraulic conductivity function, and  $C(h)$  is the water capacity function which is obtained from the soil water characteristic. The effects of rainfall rate and time distribution, initial soil

water conditions, and water table depth are incorporated as boundary and initial conditions in the solution of Equation 2-3.

Although the Richards equation provides a rather comprehensive method of determining the effects of many interactive factors on infiltration; input and computational requirements prohibits its use in DRAINMOD. The hydraulic conductivity function required in the Richards equation is difficult to measure and is available in the literature for only a few soils. Furthermore, Equation 2-3 is nonlinear and for the general case, must be solved by numerical methods requiring time increments in the order of a few seconds. The computer time required by such solutions would clearly be prohibitive for long-term simulations covering several years of record. Nevertheless, these solutions can be used to evaluate approximate methods and, in some cases, to determine parameter values required in these methods.

Approximate equations for predicting infiltration rates have been proposed by Green and Ampt (1911), Horton (1939), Philip (1957), and Holton, et al, (1967), among others. Of these, the Green-Ampt equation appears to be the most flexible and is used to characterize the infiltration component in DRAINMOD. The Green-Ampt equation was originally derived for deep homogeneous profiles with a uniform initial water content. Water is assumed to enter the soil as slug flow resulting in a sharply defined wetting front which separates a zone that has been wetted from a totally uninfiltreated zone (Figure 2-4). Direct application of Darcy's law yields,

$$f = -K_s \frac{H_2 - H_1}{L_f} \quad (2-4)$$

Where  $f$  is the infiltration rate which is equal to the downward flux (cm/hr),  $L_f$  is the length of the wetted zone,  $K_s$  is the hydraulic conductivity of the wetted or transmission zone,  $H_1$  is the hydraulic head at the soil surface and  $H_2$  is the hydraulic head at the wetting front. Taking the soil surface as the datum,  $H_1 = H_0$ , the ponded water depth and  $H_2 = h_f - L_f$  where  $h_f$  is the soil water pressure head at the wetting front. Then, Equation 2-4 may be written as,

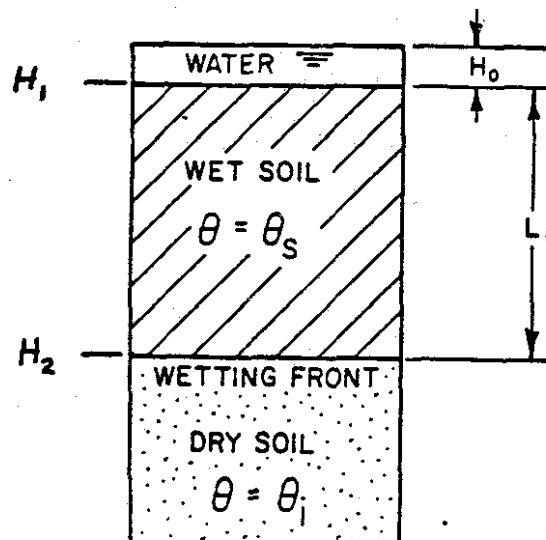


Figure 2-4. Definition sketch for derivation of the Green-Ampt equation.

$$f = -K_s (h_f - L_f - H_o) / L_f \quad (2-5)$$

Note that  $h_f$  is a negative quantity. Substituting a positive quantity,  $S_{av}$ , the effective suction at the wetting front for  $h_f$ , i.e.  $h_f = -S_{av}$  gives,

$$f = K_s (S_{av} + H_o + L_f) / L_f \quad (2-6)$$

At any time the cumulative infiltration,  $F$ , may be expressed as,  $F = (\theta - \theta_i) L_f = M L_f$ , where  $\theta$  is the volumetric water content in the wet zone,  $\theta_i$  is the initial water content and  $M$  is the initial soil water deficit (or fillable porosity). Assuming  $H_o$  is negligible compared to  $S_{av} + L_f$ , and substituting  $L_f = F/M$  into Equation 2-6 gives the Green-Ampt equation:

$$f = K_s + K_s M S_{av} / F \quad (2-7)$$

Although the original derivation by Green and Ampt assumed total saturation behind the wetting front, this requirement was in effect relaxed by Philip (1954). He assumed the water content  $\theta$ , was constant, but not necessarily equal to the total porosity. Likewise,  $K_s$  is expected to be less than the saturated hydraulic conductivity. For a given soil with a given initial water content, Equation 2-7 may be written as,

$$f = A/F + B \quad (2-8)$$

Where  $A$  and  $B$  are parameters that depend on the soil properties, initial water content and distribution, and surface conditions such as cover, crusting, etc. Note that the derivation of Equation 2-7 assumes a ponded surface so that infiltration rate is equal to infiltration capacity at all times. This is not the case for rainfall infiltration where there may be long periods of infiltration at less than the maximum rate. In this case, the infiltration rate is assumed equal to the rainfall rate until it exceeds the capacity as predicted by Equation 2-7.

In addition to uniform profiles for which it was originally derived, the Green-Ampt equation has been used with good results for profiles that become denser with depth (Childs and Bybordi, 1969) and for soils with partially sealed surfaces (Hillel and Gardner, 1970). Bouwer (1969) showed that it may also be used for nonuniform initial water contents.

Mein and Larson (1973) used the Green-Ampt equation to predict infiltration from steady rainfall. Their results were in good agreement with rates obtained from solutions to the Richards equation for a wide variety of soil types and application rates. Mein and Larson's results imply that, for uniform deep soils with constant initial water contents, the infiltration rate may be expressed in terms of cumulative infiltration,  $F$ , alone, regardless of the application rate. This was first recognized by Smith (1972) and is implicitly assumed in the use of the Green-Ampt equation to predict rainfall infiltration. Reeves and Miller (1975) extended this assumption to the case of erratic rainfall where the unsteady application rate dropped below infiltration capacity for a period of time followed by a high intensity application. Their investigations showed that the infiltra-

tion capacity could be approximated as a simple function of  $F$  regardless of the application rate versus time history. These results are extremely important for modeling efforts of the type discussed herein. If the infiltration relationship is independent of application rate, the only input parameters required are those pertaining to the necessary range of initial conditions. On the other hand, a set of parameters covering the possible range in application rates would be required for each initial condition if the infiltration relationship depends on application rate.

A frequent initial condition for shallow water table soils is an unsaturated profile in equilibrium with the water table. Solutions for the infiltration rate - time relationship for a profile initially in equilibrium with a water table 100 cm deep are given in Figure 2-5 for a sandy loam soil. The solutions were obtained by solving the Richards equation for rainfall rates varying from 2 to 10 cm/hr and for a shallow ponded surface. Note that infiltration rate is dependent on both time and the application rate (Figure 2-5). However, when infiltration rate is plotted versus cumulative infiltration,  $F = \int_0^t f \, dt$ , the relationship is nearly independent of the application rate (Figure 2-6). This is consistent with Mein and Larson's (1973) results discussed above for deep soils with uniform initial water contents.

It should be noted that resistance to air movement was neglected in predicting the infiltration relationships given in Figures 2-5 and 2-6. Such effects can be quite significant for shallow water tables where air may be entrapped between the water table and the advancing wetting front (McWhorter, 1971, 1976). Morel-Seytoux and Khanji (1974) showed that the Green-Ampt equation retained its original form when the effects of air movement were considered for deep soils with uniform initial water contents. The equation parameters were simply modified to include the effects of air movement.

Infiltration relationships for a range of water table depths are plotted in Figure 2-7 for the sandy loam considered above. Although these curves were determined from solutions to the Richards equation, similar relationships could have been measured experimentally. The parameters  $A$  and  $B$  in Equation 2-8 may be determined by using regression methods to fit the equation to observed infiltration data. The resultant parameter values will reflect the effects of air movement, as well as other factors which would have otherwise been neglected. Infiltration predictions based on such measurements will usually be more reliable than if the predictions are obtained from basic soil property measurements. Methods for determining parameters  $A$  and  $B$  from infiltration measurements and from basic soil properties are discussed in detail in Chapter 5.

The model requires inputs for infiltration in the form of a table of  $A$  and  $B$  versus water table depth. When rainfall occurs,  $A$  and  $B$  values are interpolated from the table for the appropriate water table depth at the beginning of the rainfall event. An iteration procedure is used with Equation 2-8 to determine the cumulative infiltration at the end of hourly time intervals.

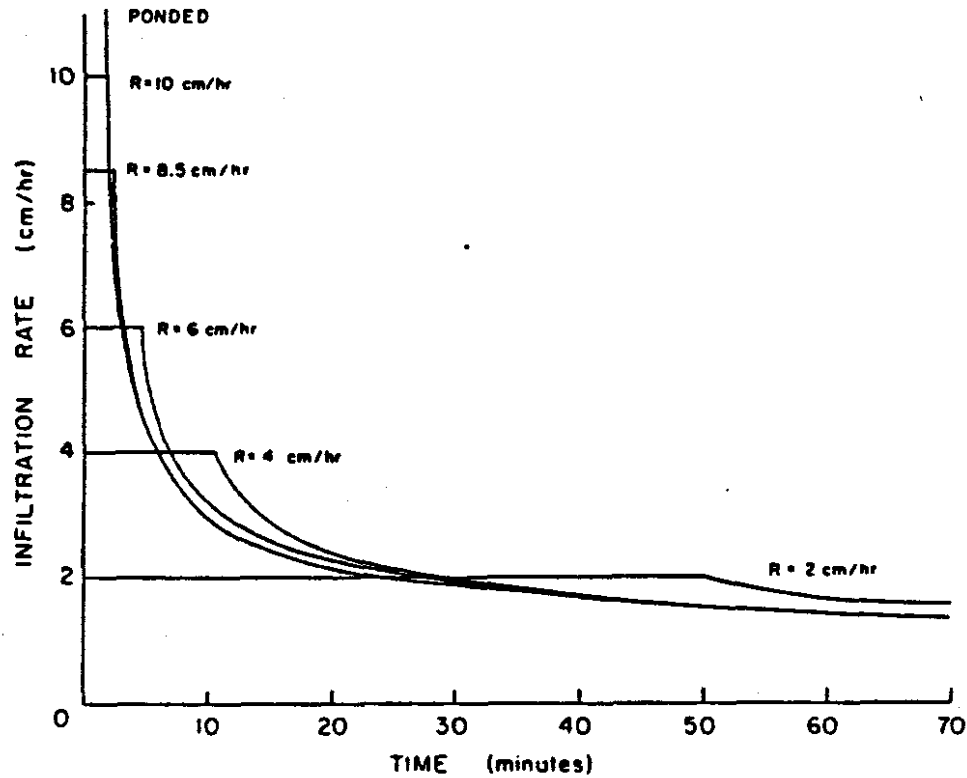


Figure 2-5. Infiltration rate versus time for a sandy loam soil initially drained to equilibrium to a water table 1.0 m deep. Note that the infiltration-time relationships are dependent on the rainfall rate.

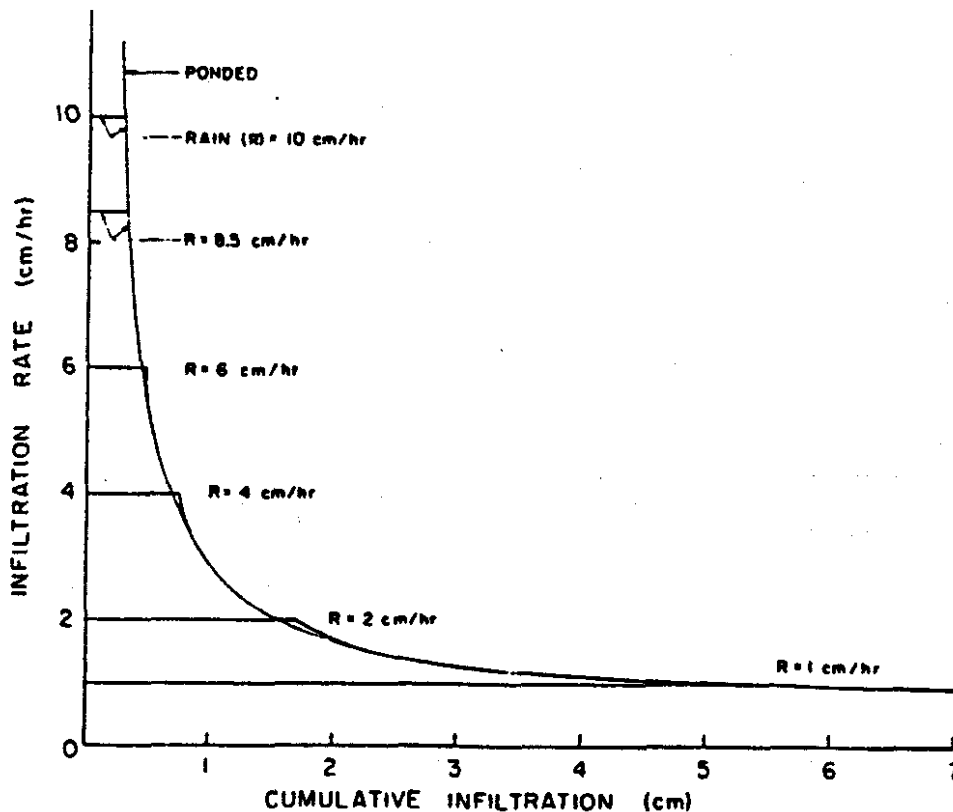


Figure 2-6. Infiltration rate - cumulative infiltration relationships as affected by rainfall rate for the same conditions as Figure 2-5.

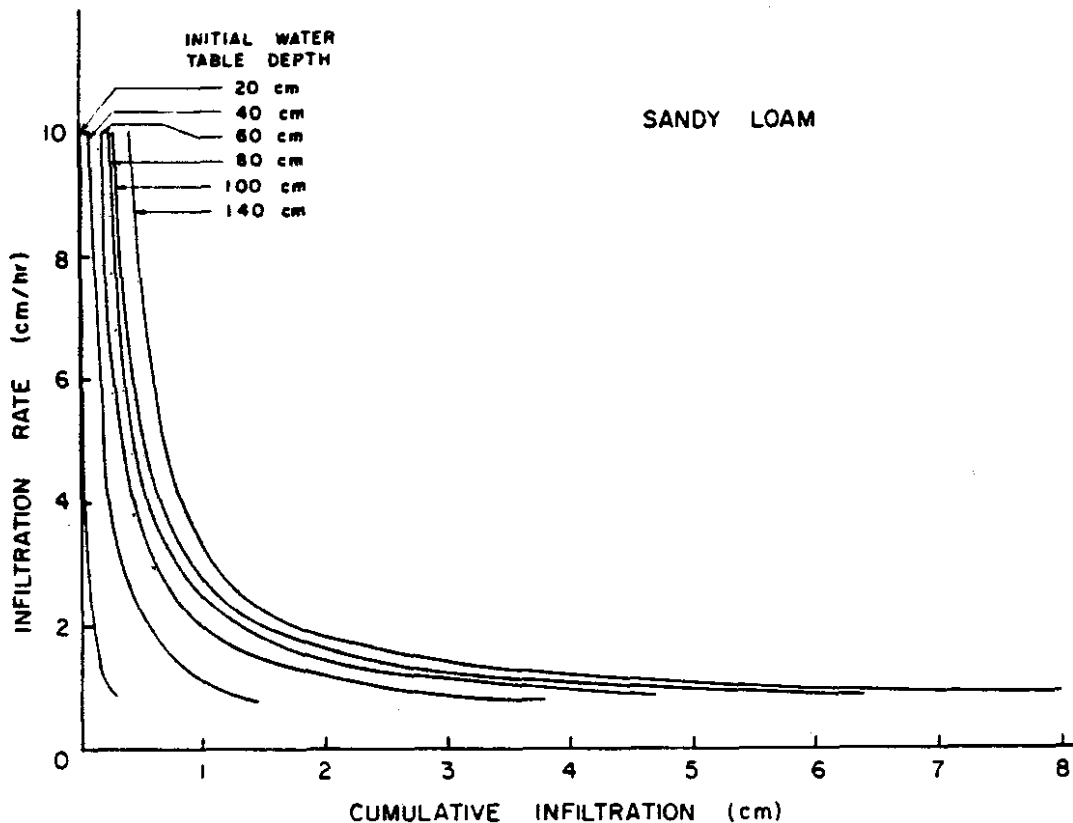


Figure 2-7. Infiltration relationships for the sandy loam soil of Figure 2-5 initially drained to equilibrium at various water table depths.

When the rainfall rate exceeds the infiltration capacity as given by Equation 2-8, Equation 2-2 is applied to conduct a water balance at the surface for  $\Delta t$  increments of 3 minutes (0.05 hour). Rainfall in excess of infiltration is accumulated as surface storage. When the surface storage depth exceeds the maximum storage depth for a given field, the additional excess is allotted to surface runoff. These values are accumulated so that, at the end of the hour, infiltration and runoff, as well as the present depth of surface storage are predicted. Hourly rainfall data are used in the program so the same procedure is repeated for the next hour using the recorded rainfall for that period. Infiltration is accumulated from hour to hour and used in Equation 2-8 until rainfall terminates and all water stored on the surface has infiltrated. Likewise, the same A and B values are used for as long as the rainfall event continues. An exception is when the water table rises to the surface, at which point A is set to  $A = 0$  and B is set equal to the sum of the drainage, ET, and deep seepage rates. An infiltration event is assumed to terminate and new A and B values obtained for succeeding events when no rainfall or surface water has been available for infiltration for a period of at least 2 hours. This time increment was selected arbitrarily and can be easily changed in the program.

Although it is assumed in the present version of the model that the A and B matrix is constant, it is possible to allow it to vary with time or to be dependent on events that affect surface cover, compaction, etc.

### Surface Drainage

Surface drainage is characterized by the average depth of depression storage that must be satisfied before runoff can begin. In most cases, it is assumed that depression storage is evenly distributed over the field. Depression storage may be further broken down into a micro component representing storage in small depressions due to surface structure and cover, and a macro component, which is due to larger surface depressions and which may be altered by land forming, grading, etc. A field study conducted by Gayle and Skaggs (1978) showed that the micro-storage component varies from about 0.1 cm for soil surfaces that have been smoothed by weathering (impacting rainfall and wind) to several centimeters for rough plowed land. Macro-storage values for eastern North Carolina fields varied from nearly 0 for fields that have been land formed and smoothed or that are naturally on grade to >3 cm for fields with numerous pot holes and depressions or which have inadequate surface outlets. Surface storage could be considered as a time dependent function or to be dependent on other events such as rainfall and the time sequence of tillage operations. Therefore, the variation in the micro-storage component during the year can be simulated. However, it is assumed to be constant in the present version of the model.

A second storage component that must be considered is the "film" or depth of surface water that is accumulated, in addition to the depression storage, before runoff from the surface begins and which remains during the runoff process. This volume is referred to as surface detention storage and depends on the rate of runoff, slope, and hydraulic roughness of the surface. It is neglected in the present version of the model which assumes that runoff moves immediately from the surface to the outlet. Actually, water that eventually runs off from one section of the field is temporarily stored as surface detention and may be infiltrated or stored at a location downslope as it moves from the field. However, the flow paths are relatively short and this volume is assumed to be small for the field size units normally considered in this model.

### Subsurface Drainage

The rate of subsurface water movement into drain tubes or ditches depends on the hydraulic conductivity of the soil, drain spacing and depth, profile depth, and water table elevation. Water moves toward drains in both the saturated and unsaturated zones and can best be quantified by solving the Richards equation for two-dimensional flow. Solutions have been obtained for drainage ditches (Skaggs and Tang, 1976), drainage in layered soils (Tang and Skaggs, 1978), and for drain tubes of various sizes (Skaggs and Tang, 1978). Input and computational requirements prohibit the use of these numerical methods in DRAINMOD, as was the case for infiltration discussed previously. However, numerical solutions provide a very useful means of evaluating approximate methods of computing drainage flux.

The method used in DRAINMOD to calculate drainage rates is based on the assumption that lateral water movement occurs mainly in the saturated region. The effective horizontal saturated hydraulic conductivity is used and the flux is evaluated in terms of the water table elevation midway between the drains and the water level or hydraulic head in the drains. Several methods are available for estimating the drain flux, including the use of numerical solutions to the Boussinesq equation. However, Hooghoudt's steady state equation, as used by Bouwer and van Schilfgaarde (1963), was selected for use in DRAINMOD. Because this equation is used for both drainage and subirrigation flux, a brief derivation is given below.

Consider steady drainage due to constant rainfall at rate,  $R$ , as shown schematically in Figure 2-8. Making the Dupuit-Forchheimer (D-F) assumptions and considering flow in the saturated zone only, the flux per unit width can be expressed as:

$$Q = -K h \frac{dh}{dx} \quad (2-9)$$

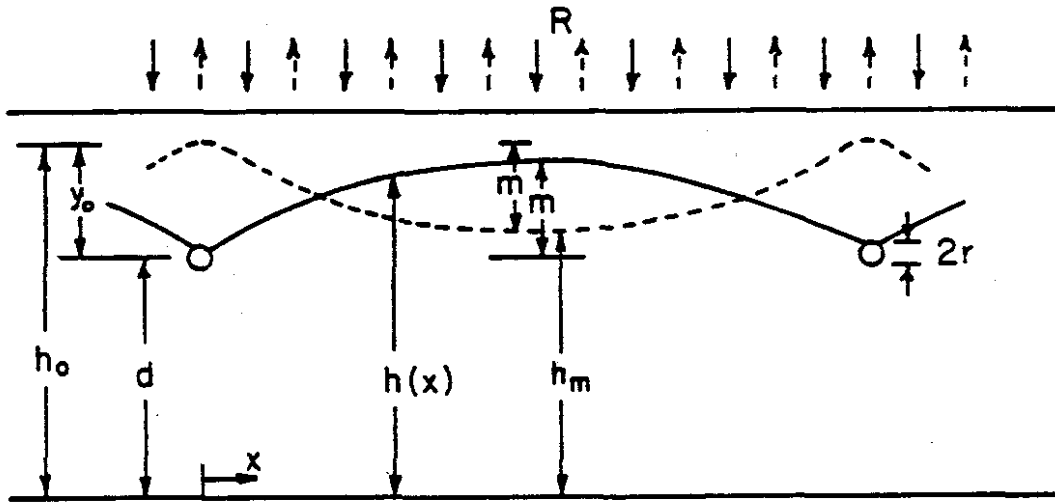


Figure 2-8. Schematic of water table drawdown to and subirrigation from parallel drain tubes.

Where  $K$  is the horizontal or lateral saturated hydraulic conductivity and  $h$  is the height of the water table above the restrictive layer. From conservation of mass we know that the flux at any point  $x$  is equal to the total rainfall between  $x$  and the midpoint,  $x = L/2$ .

$$-kh \frac{dh}{dx} = -R (L/2 - x) \quad (2-10)$$

Where the negative sign on the right-hand side of Equation 2-10 is due to the fact that flow to the drain at  $x = 0$  is in the  $-x$  direction. Separating variables and integrating Equation 2-10 subject to the boundary conditions  $h = d$  at  $x = 0$  and  $h = d + m$  at  $x = L/2$  yields an expression for  $R$  in terms of the water table elevation at the midpoint as,

$$R = \frac{4K (2md + m^2)}{L^2} \quad (2-11)$$

Although drainage is not a steady state process in most cases, a good approximation of the drainage flux can be obtained from Equation 2-11. That is, the flux resulting from a midpoint water table elevation of  $m$  may be

approximated as equal to the steady rainfall rate which would cause the same equilibrium  $m$  value. Then, the equation for drainage flux may be written as,

$$q = \frac{8 K d_e m + 4 K m^2}{C L^2} \quad (2-12)$$

Where  $q$  is the flux in cm/hr,  $m$  is the midpoint water table height above the drain,  $K$  is the effective lateral hydraulic conductivity and  $L$  is the distance between drains. Bouwer and van Schilfgaarde (1963) considered  $C$  to be equal to the ratio of the average flux between the drains to the flux midway between the drains. While it is possible to vary  $C$  depending on the water table elevation, it is assumed to be unity in the present version of the model. By solving Equation 2-12 for  $L$  with  $C = 1$ , we obtain the ellipse equation, which is often used to determine drain spacings. The ellipse equation is discussed in detail in the SCS-NEH (Section 16, Equation 4-8, and pages 4-57 to 4-69).

The equivalent depth,  $d_e$ , was substituted for  $d$  in Equation 2-11 in order to correct for convergence near the drains. The D-F assumptions used in deriving Equation 2-12 imply that equipotential lines are vertical and streamlines horizontal within the saturated zone. Numerical solutions for the hydraulic head (potential) distribution and water table position are plotted in Figure 2-9 for four different drains: a conventional 114 mm O.D. drain tube, a 114 mm tube with open side walls, an open ditch, and a drain tube surrounded by a square envelope, 0.5 m x 0.5 m in cross-section. The solutions were obtained by solving the two-dimensional Richards equation which requires no simplifying assumptions. These solutions show that, except for the region close to the drain, the equipotential lines in the saturated zone are nearly vertical. Thus, the D-F assumptions would appear reasonable for this case, providing convergence near the drain can be accounted for.

Hooghoudt (van Schilfgaarde, 1974) characterized flow to cylindrical drains by considering radial flow in the region near the drains and applying the D-F assumptions to the region away from the drains. The Hooghoudt analysis has been widely used to determine an equivalent depth,  $d_e$ , which, when substituted for  $d$  in Figure 2-8 will tend to correct drainage<sup>e</sup> fluxes predicted by Equation 2-12 for convergence near the drain. Moody (1967) examined Hooghoudt's solutions and presented the following equations from which  $d_e$  can be obtained.

For  $0 < d/L < 0.3$

$$d_e = \frac{d}{1 + \frac{d}{L} \left\{ \frac{8}{\pi} \ln \left( \frac{d}{r} \right) - \alpha \right\}} \quad (2-13)$$

In which

$$\alpha = 3.55 - \frac{1.6d}{L} + 2 \left( \frac{2}{L} \right)^2 \quad (2-14)$$

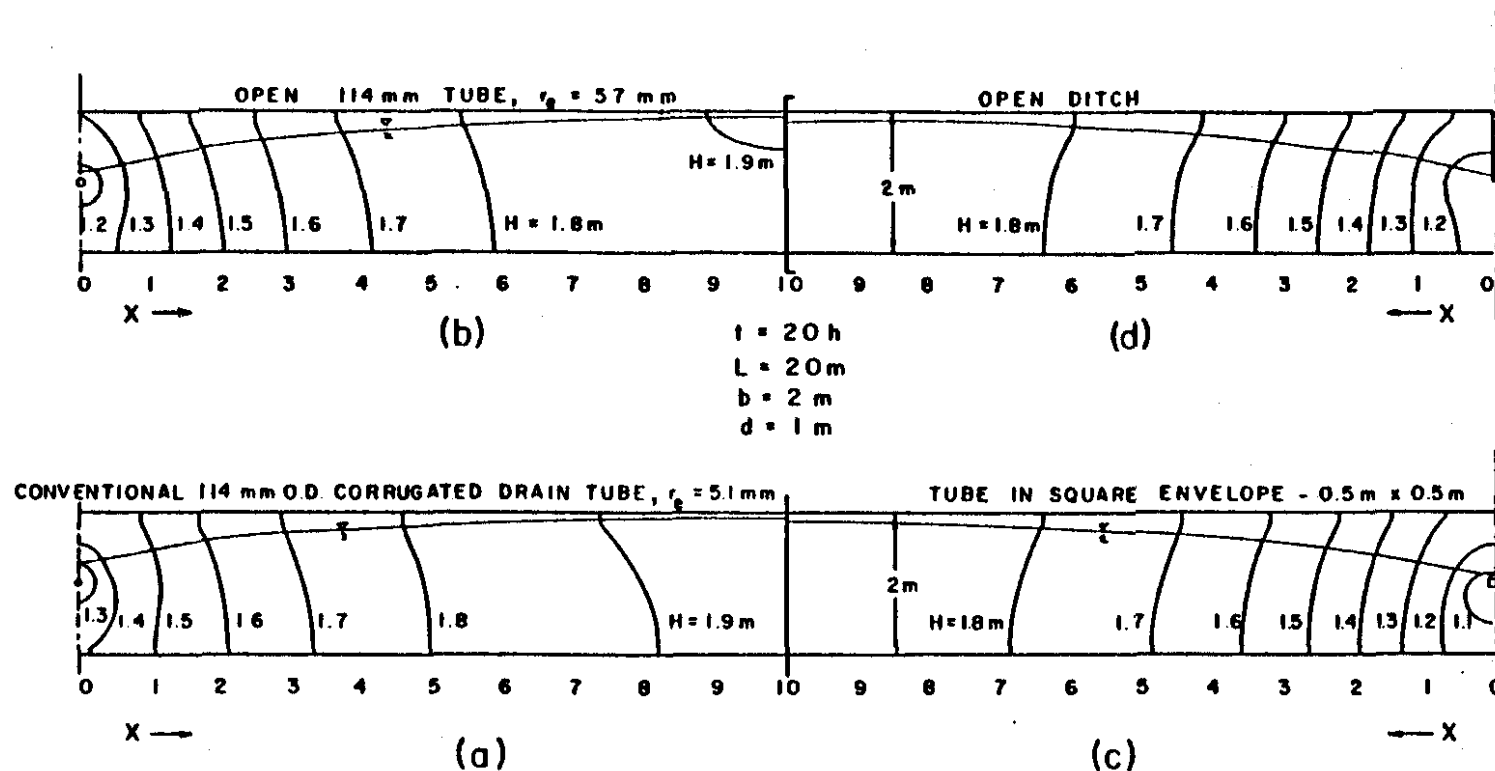


Figure 2-9. Water table position and hydraulic head,  $H$ , distribution in a Panoche soil after 20 hours of drainage to (a) conventional 114 mm (4-inch) drain tubes; (b) wide open (no walls) 114 mm diameter drain tubes; (c) a drain tube in a square envelope 0.5 m x 0.5 m; and (d) an open ditch 0.5 m wide. The drain spacings in all cases were 20 m. (After Skaggs and Tang, 1978).

And for  $d/L > 0.3$

$$d_e = \frac{L\pi}{8 \left\{ \ln \left( \frac{L}{r} \right) - 1.15 \right\}} \quad (2-15)$$

In which  $r$  = drain tube radius. Usually  $\alpha$  can be approximated as  $\alpha = 3.4$  with negligible error for design purposes.

For real, rather than completely open drain tubes, there is an additional loss of hydraulic head due to convergence as water approaches the finite number of openings in the tube. The effect of various opening sizes and configurations can be approximated by defining an effective drain tube radius,  $r$ , such that a completely open drain tube with radius  $r_e$  will offer the same resistance to inflow as a real tube with radius  $r$ . Dennis and Trafford (1975) used Kirkham's (1949) equation for drainage from a ponded surface and measured drain discharge rates in a laboratory soil tank to define effective drain tube radii. Bravo and Schwab (1977) used an electric analog model to determine the effect of openings on radial flow to corrugated drain tubes. Their data were used by the author (Skaggs, 1978b) to determine  $r = 0.51$  cm for 11.4 cm (4.5-in.) O.D. tubing. Standard 4-in. (100-cm) corrugated tubing has an outside diameter of approximately 4.5 in. The same methods are used to determine  $r$  and then  $d_e$  which is an input to the model. More discussion of entrance resistance into drain is given in the FAO Irrigation and Drainage Paper No. 9 (FAO, 1972).

The above discussion treats the soil as a homogeneous media with saturated conductivity  $K$ . Most soils are actually layered with each layer having a different  $K$  value. Since subsurface water movement to drain is primarily in the lateral direction, the effective hydraulic conductivity in the lateral direction is used in Equation 2-12. Referring to Figure 2-10, the equivalent conductivity is calculated using the equation,

$$K_e = \frac{K_1 d_1 + K_2 D_2 + K_3 D_3 + K_4 D_4}{d_1 + D_2 + D_3 + D_4} \quad (2-16)$$

Because the thickness of the saturated zone in the upper layer is dependent on the water table position,  $K_e$  is determined prior to every flux calculation using the value of  $d_1$  which depends on the water table position. If the water table is below layer 1,  $d_1 = 0$  and a similarly defined  $d_2$  is substituted for  $D_2$  in Equation 2-16.

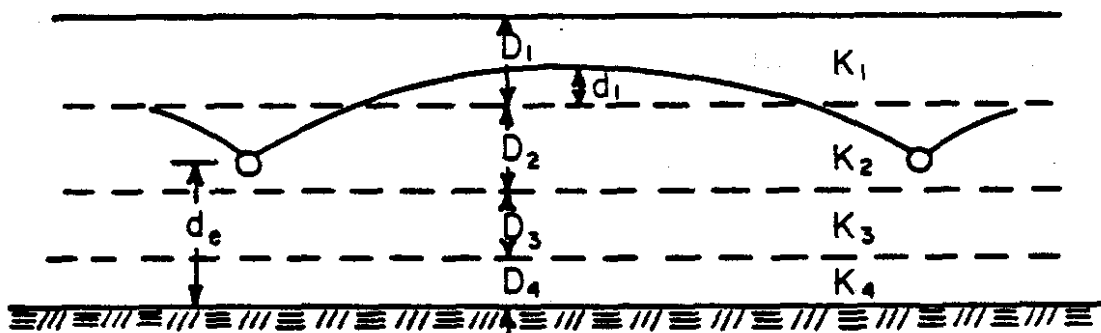


Figure 2-10. Equivalent lateral hydraulic conductivity is determined for soil profiles with up to 5 layers.

The use of the approach discussed above, employing Equations 2-12 through 2-16, will give satisfactory results as long as there are not major differences in the conductivities of the individual layers. When major differences occur, the thicknesses and conductivities of the layers should be considered in defining the equivalent depth,  $d_e$ . Van Beers (1976) summarized methods for predicting drain flux which consider convergence to the drains and layered profiles. These steady state methods included that developed by Ernst, which divides the loss in hydraulic head ( $m$  in Figure 2-8) into three components:  $m = h_v + h_h + h_r$  where  $h_v$  = head loss due to vertical flow,  $h_h$  = head loss due to horizontal flow and  $h_r$  = head loss due to radial flow near the drain. This approach was combined with that of Hooghoudt to give the Hooghoudt-Ernst equation, which does not require a separate calculation for  $d_e$ . However, it is necessary to determine a geometric factor from a nomograph for some layered systems. The modified Hooghoudt-Ernst equation is also discussed by van Beers (1976) and could be easily employed in DRAINMOD.

The discussed methods above for predicting drainage flux assumed a curved (elliptical) water table completely below the soil surface, except at the midpoint where it may be coincident with the surface. However, in some cases, the water table may rise to completely inundate the surface with ponded water remaining there for relatively long periods of time. Then, the D-F assumptions will not hold as the streamlines will be concentrated near the drains with most of the water entering the soil surface in that vicinity. Kirkham (1957) showed that in one case, more than 95 percent of the flow entered the surface in a region bounded by  $\pm$  one-quarter of the drain spacing. The shape of the streamlines for drainage from a ponded surface as compared to that for water table drawdown is shown in Figure 2-11. Drainage flux for a ponded surface can be quantified using an equation derived by Kirkham (1957):

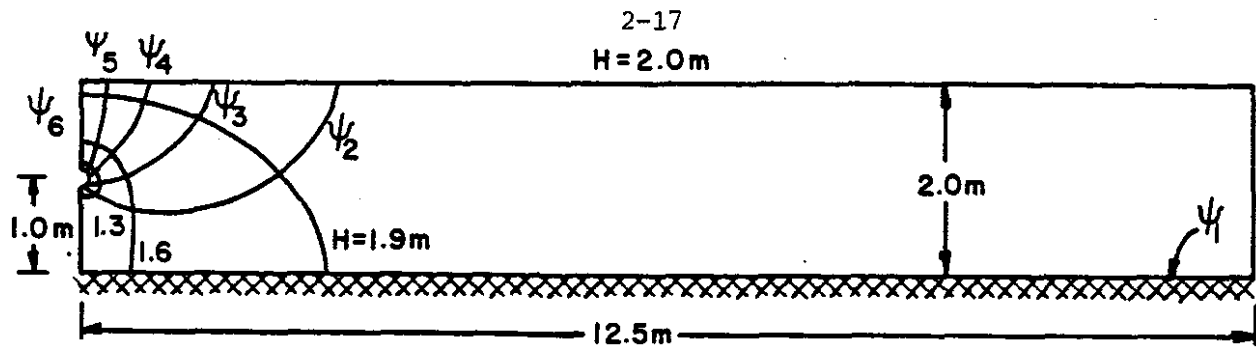
$$q = \frac{4\pi k (t + b - r)}{gL} \quad (2-17)$$

Where

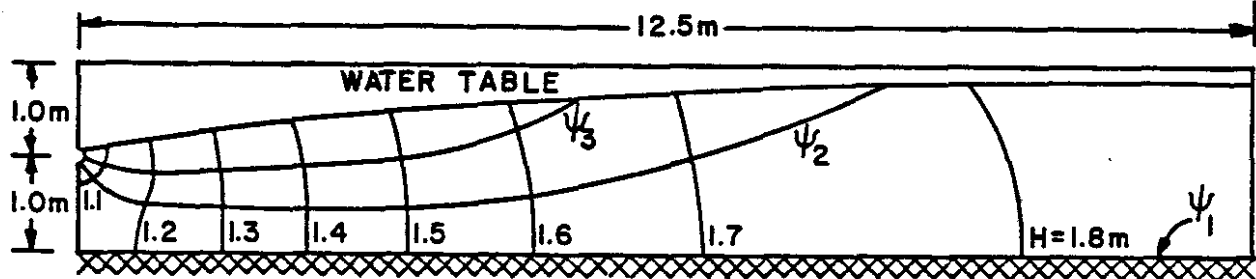
$$g = 2 \ln \left[ \frac{\tan(\pi(2d-r)/4h)}{\tan \pi r/4h} \right] + 2 \sum_{m=1}^{\infty} \ln \left[ \frac{\cosh(\pi m L/2h) + \cos(\pi r/2h)}{\cosh(\pi m L/2h) - \cos(\pi r/2h)} \right] \cdot \frac{\cosh(\pi m L/2h) - \cos(\pi(2d-r)/2h)}{\cosh(\pi m L/2h) + \cos(\pi(2d-r)/2h)} \quad (2-18)$$

Where  $h$  is the depth of the profile (Figure 2-12) - actual depth not equivalent depth.

Equation 2-17 can be used after the water table rises to the surface for as long as surface water can move freely toward the drains. Recall that water is stored on the surface in depressions, so movement overland toward the drains may be restricted by surface roughness as shown schematically in Figure 2-12. When rows are oriented perpendicular to the drain tube direction, water may move along the furrows to the region above the drains, but still remain in lower depressional areas (with a maximum depth of  $S$ , as shown in Figure 2-12). When the ponded depth becomes less than  $S_1$ , water can no longer move freely over the surface, the depth of water ponded over



(a) DRAINAGE FROM A PONDED SURFACE



(b) DRAINAGE DURING WATER TABLE DRAWDOWN

Figure 2-11. Equipotential  $H$  and streamlines  $\Psi$  for drainage from ponded surface and for drainage during water-table drawdown.

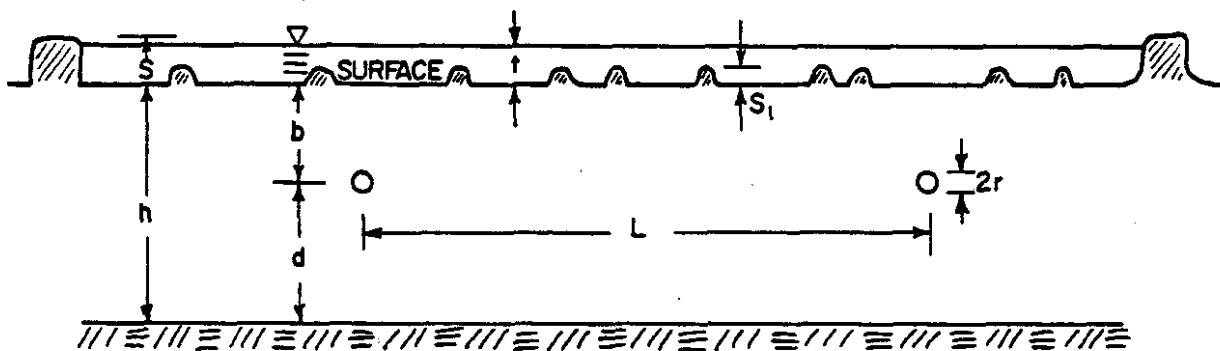


Figure 2-12. Schematic of drainage from a ponded surface. Water will move over the surface to the vicinity of the drains until the ponded depth becomes less than  $S_1$ . The maximum depressional storage is  $S$ .

the drains will decrease more rapidly than that near the midpoint and Equation 2-12 will provide a better estimate for drainage flux than will 2-17.

Use of equations 2-12 and 2-17 assume that drainage is limited by the rate of soil water movement to the lateral drains and not by the hydraulic capacity of the drain tubes or of the outlet. Usually, the sizes of the drain tubes are chosen to provide a design flow capacity, which is called the drainage coefficient, D.C. Typically, the D.C. may be 1 to 2 cm per day (about 3/8 to 3/4 inches per day) depending on the geographic location and crops to be grown. The D.C. for a given slope and size of drain (either lateral or main) can be obtained from the N.E.H. Section 16, Figures 4-36, or by direct use of the Manning equation. When the flux given by equations 2-12 or 2-17 exceeds the D.C.,  $q$  is set equal to the D.C. in DRAINMOD as suggested by Chieng, et al, (1978). The water level in the main outlet (canal or river) may also limit the drainage flux in certain cases. However, the outlet water level is affected by surface and subsurface drainage from a much larger area than the field size areas analyzed in DRAINMOD. Such outlet limitations would depend on both the site and storm event and are not treated in the present version of DRAINMOD. That is, the outlet capacity is assumed to be adequate to carry the drainage and runoff from the fields.

In summary, the drainage flux should be calculated using a three-step approach as follows:

1. For water tables below the surface and for ponded depths  $< S_1$ , use Equation 2-12.
2. For ponded depths  $> S_1$ , use Equation 2-17.
3. When the flux predicted by the appropriate equation, either 2-12 or 2-17, is greater than the D.C., set the flux equal to the D.C.

### Subirrigation

When subirrigation is used, water is raised in the drainage outlet so as to maintain a pressure head at the drain of  $h_o$  (refer to the broken curve in Figure 2-8). If the boundary condition  $h = h_o$  at  $x = 0$  is used in solving Equation 2-10, the equation corresponding to Equation 2-12 for flux is,

$$q = \frac{4K}{L^2} (2 h_o m + m^2) \quad (2-19)$$

Where  $m$  is always defined as water table elevation midway between the drains minus the water table elevation at the drain,  $(h_m - h_o)$ , in this case (Figure 2-8). To correct for convergence,  $h_o = y_o + d_e$  is the equivalent water table elevation at the drain and  $h_m$  is the equivalent water table elevation midway between the drains. For subirrigation,  $h_o > h_m$  and both  $m$

and  $q$  are negative. Convergence losses, at the drain, are treated in the same manner as in drainage by using the equivalent depth to the impermeable layer,  $d_e$ , rather than the actual depth,  $d$ , to define  $h_o$  in equation 2-19. Equation 2-19 was derived by making the D-F assumptions and solving the resulting flow equation for steady evaporation from the field surface at rate  $q$ . The magnitude of  $q$  increases as  $m$  becomes more negative, i.e., as  $h_m$  becomes smaller, until the water table at the midpoint reaches the equivalent depth of the impermeable layer,  $h_m = 0$ . For deeper midpoint water table depths, which can occur because the actual depth to the impermeable layer is deeper than the equivalent depth, equation 2-19 predicts a decrease in the magnitude of  $q$ . Ernst (1975) observed that this is inconsistent with the physics of flow since the maximum subirrigation rate should occur when the midpoint water table reaches the impermeable layer. He derived an equation similar to Equation 2-19 to correct these deficiencies. The equation may be written in the present notation as,

$$q = \frac{4K m (2h_o + \frac{h_o}{D_o} m)}{L^2} \quad (2-20)$$

Where  $D_o = y_o + d$ ,  $d$  is the distance from the drain to the impermeable layer, and  $h_o$  is the same as defined previously,  $h_o = y_o + d_e$ . Equation 2-20 is now used in DRAINMOD to predict subirrigation flux.

When controlled drainage is used, a weir is set at a given elevation in the drainage outlet. The actual water level in the drain is not fixed as it is with subirrigation, but depends on size of the outlet, previous drainage, etc. If the water table elevation in the field is higher than the water level in the drain, drainage will occur and the water level in the drain will increase. If it rises to the weir level, additional drainage water will spill over the weir and leave the system. When the water table in the field is lower than that in the drain, water will move into the field at a rate given by Equation 2-10 raising the water table in the field or supplying ET demands while reducing the water level in the drain. The amount of water stored in the drainage outlet and the water level in the outlet during subirrigation or controlled drainage is computed at each time increment by a DRAINMOD subroutine called YDITCH. This subroutine uses the geometry of the outlet, weir setting and drainage or subirrigation flux to determine the water level in the outlet at all times.

### Evapotranspiration

The determination of evapotranspiration (ET) is a two-step process in the model. First, the daily potential evapotranspiration (PET) is calculated in terms of atmospheric data and is distributed on an hourly basis. The PET represents the maximum amount of water that will leave the soil system by evapotranspiration when there is a sufficient supply of soil water. The present version of the model distributes the PET at a uniform rate for the 12 hours between 6:00 a.m. and 6:00 p.m. In case of rainfall, hourly PET is set equal to zero for any hour in which rainfall occurs. After PET is calculated, checks are made to determine if ET is limited by

soil water conditions. If soil water conditions are not limiting, ET is set equal to PET. When PET is higher than the amount of water that can be supplied from the soil system, ET is set equal to the smaller amount. Methods used for determining PET and the rate that water can be supplied from the soil water system are discussed below.

Potential ET depends on climatological factors which include net radiation, temperature, humidity, and wind velocity. Evapotranspiration can be directly measured with lysimeters or from water balance-soil water depletion methods. However, such measurements are rarely available for a given time and location and most PET values are obtained from climatological data using one of the many prediction methods. Jensen (1973) presented a thorough review of the consumptive use of water. He included detailed discussion and summary of the theory of evaporation and evapotranspiration (ET); engineering requirements for ET data; sources of ET data; evaluation of methods for estimating ET and utilization of ET data. Methods for predicting PET in humid regions were reviewed by McGuinness and Borden (1972) and Mohammad (1978). A summary of some of the methods, including required climatological input data is given in Table 2-1. Perhaps the most reliable method is the one developed by Penman (1948, 1956) which is based on an energy balance at the surface. The method requires net radiation, relative humidity, temperature, and wind speed, as input data. Additional methods that could be used include, among others, those by Jensen, et al, (1963), Stephens and Stewart (1963), Turc (1961), and van Bavel (1961). However, all of these equations require daily solar or net radiation as input data and such data are available for only very few locations. Because we are interested in conducting simulations in many locations throughout the United States, it is necessary to estimate ET based on readily available input data.

The method selected for use in the model was the empirical method developed by Thornthwaite (1948). He expressed the monthly PET as,

$$e_j = c \bar{T}_j^a \quad (2-21)$$

Where  $e_j$  is the PET for month  $j$  and  $\bar{T}_j$  is the monthly mean temperature ( $^{\circ}\text{C}$ ),  $c$  and  $a$  are constants which depend on location and temperatures. The coefficients  $a$  and  $c$  are calculated from the annual heat index,  $I$ , which is the sum of the monthly heat indexes,  $i_j$ , given by the equation,

$$i_j = (\bar{T}_j/5)^{1.514} \quad (2-22)$$

$$I = \sum_{j=1}^{12} i_j \quad (2-23)$$

The heat index is computed from temperature records and the monthly PET calculated from Equation 2-21. Then, the monthly PET value is corrected for number of days in the month and the number of hours between sunrise and

Table 2-1. Summary of PET prediction methods for humid regions (from Mohammad, 1978).

| Method             | Climatological Factors |    |    |    |   |   |                |                |    |    |   |    |    |                                                                                                                                | Formula Used |
|--------------------|------------------------|----|----|----|---|---|----------------|----------------|----|----|---|----|----|--------------------------------------------------------------------------------------------------------------------------------|--------------|
|                    | TC                     | TA | RH | RI | H | U | e <sub>s</sub> | e <sub>d</sub> | DL | RT | S | PT | PD |                                                                                                                                |              |
| Penman             |                        | ✓  | ✓  |    | ✓ | ✓ | ✓              | ✓              |    |    |   | ✓  |    | PET = (ΔH + E <sub>a</sub> γ)/(Δ+γ) in mm/day                                                                                  |              |
| Jensen-Haise       |                        | ✓  |    | ✓  |   |   |                |                |    |    |   |    |    | PET = [0.014(TA)-0.37]RI(0.000673 in in/day                                                                                    |              |
| Stephens & Stewart |                        | ✓  |    | ✓  |   |   |                |                |    |    |   |    |    | PET = (0.0082 TA - 0.19)(RI/1500) in in/day                                                                                    |              |
| Turc               | ✓                      |    |    | ✓  |   |   |                |                |    |    |   |    |    | PET = 0.40 TC(RI + 50)/(TC + 15) in in/day                                                                                     |              |
| Grassi             |                        | ✓  |    | ✓  |   |   |                |                |    |    |   |    |    | PET = K <sub>RS</sub> C <sub>T</sub> C <sub>crc</sub> F in in/day                                                              |              |
| Thornthwaite       | ✓                      |    |    |    |   |   |                |                |    |    |   |    |    | PET = 1.6 (10 TC/I) <sup>a</sup> in cm/month                                                                                   |              |
| Blaney-Criddle     |                        | ✓  |    |    |   |   |                |                | ✓  |    |   |    |    | PET = (0.0173 TA - 0.314)KC x TA(DL/4465.5) in in/day                                                                          |              |
| Hamon              |                        |    |    |    |   |   |                |                |    |    | ✓ | ✓  |    | PET = C S <sup>2</sup> PT/100 in in/day                                                                                        |              |
| Papadakis          |                        |    |    |    |   |   | ✓              | ✓              |    |    |   |    |    | PET = 0.5625 (e <sub>a</sub> - e <sub>d-2</sub> ) in cm/month                                                                  |              |
| Makkink            | ✓                      |    |    | ✓  |   |   |                |                |    |    |   |    |    | PET = 0.61 RI[Δ/(Δ+γ)] - 0.12 in mm/month                                                                                      |              |
| Christiansen       |                        | ✓  | ✓  |    |   | ✓ |                |                |    | ✓  | ✓ |    |    | PET = 0.473 R <sub>T</sub> C <sub>T</sub> C <sub>w</sub> C <sub>H</sub> C <sub>S</sub> C <sub>E</sub> C <sub>M</sub> in in/day |              |
| van Bavel          |                        | ✓  |    |    | ✓ | ✓ |                |                |    |    |   |    | ✓  | PET = [(Δ/γ)(H/L) + BV PD]/[(Δ/γ) + 1] in in/day                                                                               |              |

PET = Potential evapotranspiration

TC = Mean air temperature in °C

TA = Mean air temperature in °F

RH = Relative humidity

RI = Solar radiation in langley

H = Net radiation in langley

U = Wind speed at a height of 2 meters

e<sub>s</sub> = Saturated vapor pressure of the air in mm mercury

e<sub>d</sub> = Actual vapor pressure of the air in mm mercury

DL = Day length in hours

L = Latent heat of vaporization of water

RT = Solar radiation at the top of the atmosphere in inches of evaporation equivalent

S = Possible hours of sunshine in units of 12 hours

PT = Saturated vapor density

PD = Vapor pressure deficit in mm

K = Constant (0.537)

C<sub>crc</sub> = Plant cover coefficient (for meadow is 1.0)

F = Constant (for alfalfa is 1.09)

KC = Crop growth stage coefficient

C = Constant (0.55)

C<sub>E</sub> = Coefficient for the elevation of the site

C<sub>M</sub> = Monthly vegetative coefficient

a =  $6.75 \times 10^{-7} I^3 - 7.71 \times 10^{-5} I^2 + 1.792 \times 10^2 I + 0.4924$

Δ = Slope of saturated vapor pressure curve.

sunset in the day by adjusting for the month and latitude. Daily values may be obtained from the monthly PET by using the daily mean temperature according to the methods given by Thornthwaite and Mather (1957).

The PET is computed in the main program of DRAINMOD from recorded daily maximum and minimum temperature values. The heat index must be determined and entered, along with the latitude of the site, separately. Adjustments for day length and number of days in the month are made in the program based on latitude and date. This version of the main program also inputs hourly rainfall from climatological records and is used for long-term simulations. Another version of the main program was developed to input climatological data obtained in experiments to test the model. The daily PET values were calculated separately and read into the model from cards. In this case, any method could be used to determine PET, although the Thornthwaite method was still used for all tests.

The approximate nature of the Thornthwaite equation for predicting daily PET should be emphasized. The following comments on the method were made by Taylor and Ashcroft (1972):

"This equation, being based entirely upon a temperature relationship, has the disadvantage of a rather flimsy physical basis and has only weak theoretical justification. Since temperature and vapor pressure gradients are modified by the movement of air and by the heating of the soil and surroundings, the formula is not generally valid, but must be tested empirically whenever the climate is appreciably different from areas in which it has been tested. ... In spite of these shortcomings, the method had been widely used. Because it is based entirely on temperature data that are available in a large number of localities, it can be applied in situations where the basic data of the Penman method are not available."

Several of the methods listed in Table 2-1, as well as others not listed, will give more accurate estimates of PET than Thornthwaite. The Penman (1948) equation and the combination method by van Bavel (1966) are reliable methods, but require input data that are not available for many locations, especially for the long, continuous period of record needed in application of DRAINMOD. However, it is important to note that, if the input data can be obtained, these or other methods can be used in DRAINMOD by simply substituting for the Thornthwaite method in the main program. The necessary data for other methods may be available for some locations and it may be desirable to change the PET component for such applications. Measurements of net radiation, wind speed, RH, etc., are presently being conducted, analyzed and stored using modern micro computer technology. Thus, complete sets of required input data for the more sophisticated PET prediction equations may be available for many locations in the future.

In spite of the deficiency of the Thornthwaite method, it has given good results in some areas and it appears to be sufficiently accurate for drainage modeling in humid regions. Mohammad (1978) compared six methods for predicting PET for eastern North Carolina conditions. His study was closely associated with North Carolina State University experiments to test

DRAINMOD. Mohammad found that the PET values predicted by the Thornthwaite method were somewhat higher than that predicted from pan evaporation measurements and lower than predictions from the Penman method. Considering the difference in input requirements, the Thornthwaite method appears to provide an acceptable estimate of PET for North Carolina conditions.

An alternative method of estimating PET is to use measured daily pan evaporation corrected by a pan coefficient. The pan coefficient is usually taken to be about 0.7. Daily pan evaporation values can easily be read into DRAINMOD, if they are available. This method is reliable for a wider range of locations and conditions than the Thornthwaite method. The problem with its use is that the data may not be available for locations of interest.

Another method for estimating ET in terms of temperature and day length is the Blaney-Criddle formula. This method was developed by Blaney and Criddle (1947) for irrigated regions of the United States. The method has been modified by the SCS and is described, in detail, along with charts for consumptive-use and crop growth stage coefficients in Technical Release No. 21, "Irrigation Water Requirements." The Blaney-Criddle methods has been widely correlated with field experiments having been empirically developed for irrigated areas of the semi-arid and arid regions. According to Taylor and Ashcroft (1972), the method gives an estimate of actual ET, rather than PET, because it is based on correlations with existing irrigation practice. This would cause some difficulty in using the Blaney-Criddle method in DRAINMOD where the effect of limiting soil water conditions is considered separately from PET calculations. Taylor and Ashcroft state that the method "is probably adequate for many estimates of seasonal ET under conditions similar to those for which crop coefficients and consumptive use factors have been determined. It has not proven reliable for shorter periods." Still, this may be a suitable alternative to the Thornthwaite method, especially for applications in the west, although it would require some modification of DRAINMOD.

Each ET calculation involves a check to determine if soil water conditions are limiting. When the water table is near the surface or when the upper layers of the soil profile have a high water content, ET will be equal to PET. However, for deep water tables and drier conditions, ET may be limited by the rate that water can be taken up by plant roots. Gardner (1958) analyzed the factors controlling steady evaporation from soils with shallow water tables by solving the governing equations for unsaturated upward water movement. For soils with a given functional relationship between unsaturated hydraulic conductivity and pressure head,  $K = K(h)$ , Gardner presented simplified expressions for the maximum evaporation rate in terms of water table depth and the conductivity function parameters. For steady unsaturated flow, the upward flux is constant everywhere and the governing equation may be written as,

$$\frac{d}{dz} \left[ K(h) \frac{dh}{dz} - K(h) \right] = 0 \quad (2-24)$$

Where  $h$  is the soil water pressure head and  $z$  is measured downward from the surface (Figure 2-13). For any given water table depth, the rate of upward water movement will increase with soil water suction ( $-h$ ) at the surface. Therefore, the maximum evaporation rate for a given water table

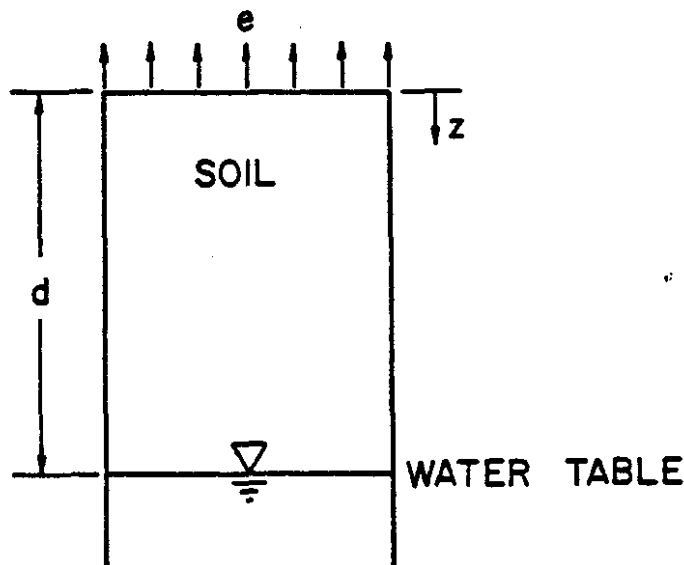


Figure 2-13. Schematic for upward water movement from a water table due to evaporation.

depth can be approximated by solving Equation 2-24, subject to a large negative  $h$  value, say  $h = -1000$  cm, at the surface ( $z = 0$ ) and  $h = 0$  at  $z = d$ , the water table depth. Numerical solutions to Equation 2-24 can be obtained for layered soils and for functional or tabulated  $K(h)$  relationships (See Chapter 5 and Appendix F). By obtaining solutions for a range of water table depths, the relationship between maximum rate of upward water movement and water table depth can be developed. Such a relationship is shown in Figure 2-14 for the Wagram loamy sand studied by Wells and Skaggs (1976).

Relationships such as that shown in Figure 2-14 are read as inputs to the model in tabular form. Then, if the PET is 5 mm/day, the ET demand could be satisfied directly from the water table for water table depths less than about 0.64 m. For deeper water tables, ET for that day would be less than 5 mm or the difference would have to be extracted from root zone storage. The root depth will be discussed in a later section. However, it should be pointed out that the roots are assumed to be concentrated within an effective root zone, and that the surface boundary condition may be shifted to the bottom of the root zone, as indicated by the abscissa label in Figure 2-14.

Methods used for determining whether ET is limited by soil water conditions can best be described by an example. Assume that the Wagram soil shown in Figure 2-14, the water table at the beginning of day  $k$  is 0.91 m between the bottom of the dry zone; the root zone depth is 10 cm and PET for

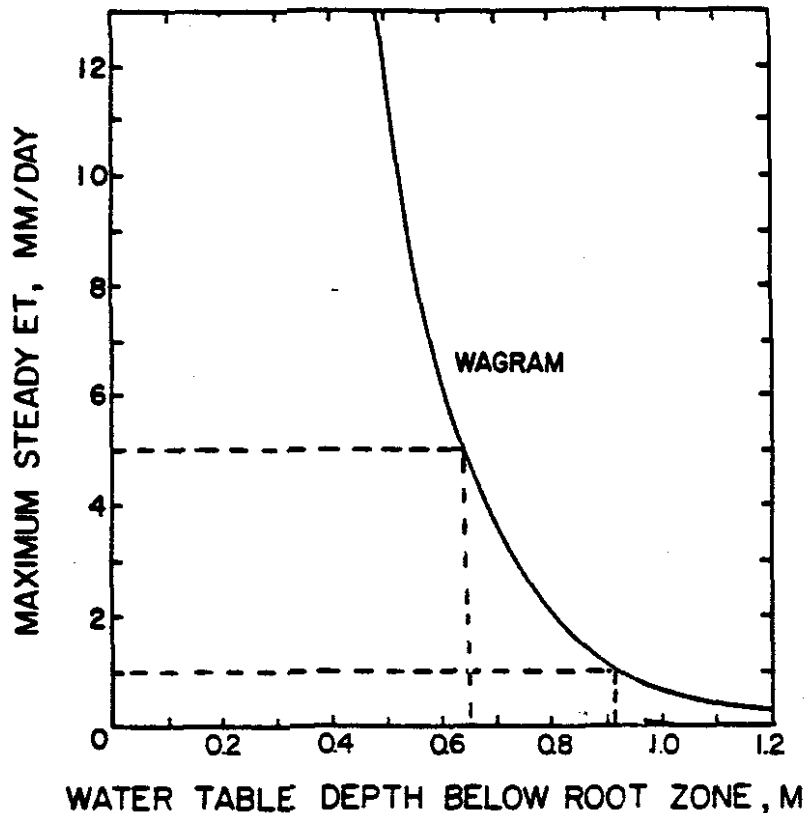


Figure 2-14. Relationship between maximum rate of upward water movement versus water table depth below the root zone for a Wagram loamy sand.

day  $x$  is 5 mm. From Figure 2-14, we find that 1 mm of the PET demand will be supplied from the water table, leaving a 4 mm deficit. This deficit can be supplied by water stored in the root zone, if it has not already been used up. Here it is assumed that the plant roots will extract water down to some lower limit water content,  $\theta_{ll}$ ; the wilting point water content has been used for  $\theta_{ll}$ , but a larger value can be substituted if desired. For convenience, this water is assumed to be removed from a layer of soil starting at the surface and creating a dry zone which is limited to a maximum depth equal to the rooting depth. Taking a value of  $\theta_{ll}$  of 0.15 and a saturated water content,  $\theta_s$ , of 0.35 the 4 mm deficit would dry out a layer of thickness  $0.4 \text{ cm} / (0.35 - 0.15) = 2 \text{ cm}$ . Thus, the dry zone depth at the end of day  $k$ , would be increased by 2 cm. Further, the total water table depth would be increased by 2 cm in addition to the increase resulting from the upward movement of the 1 mm of water. Under these conditions, ET for day  $k$  will be equal to the PET for 5 mm. When the dry zone depth becomes equal to the rooting depth, ET is limited by soil water conditions and is set equal to the upward water movement. For example, if the dry zone at the beginning of day  $k$  was already 10 cm deep, the ET for day  $k$  would be limited to the rate of upward water movement of 1 mm, rather than 5 mm. The storage

volume in the dry zone is accumulated separately from the rest of the unsaturated zone. It is updated on a day-to-day, hour-to-hour basis, and is assumed to be the first volume filled when rainfall or irrigation occurs.

One problem with the use of the methods discussed above for calculating ET, is the difficulty of obtaining reliable  $K(h)$  data needed to determine the relationship given in Figure 2-14 for many field soils. This is particularly true for multilayered soils and is discussed in detail in Chapter 5. A more approximate method was developed and may be used as an option in the model by estimating a single critical or limiting depth parameter. When this option is used, it is assumed that the potential ET rate will be supplied from the water table until the distance between the root zone and the water table becomes greater than the limiting depth. After the distance from the root zone and the water table reaches the limiting depth, it is assumed that water will be extracted from the root zone at a rate still equal to the potential ET rate, until the root zone water content reaches  $\theta_{ll}$  in the same manner as was explained above when PET was greater than the rate of upward water movement. Thus, water is removed from the root zone from the surface downward until the depth of the resulting dry zone is equal to the rooting depth. Then, ET is assumed equal to zero. This option is considered more approximate than the alternative method and should be used only when the relationship between maximum upward flux and water table depth cannot be obtained.

Predictions of ET, as limited by soil water conditions, are shown schematically in Figure 2-15 for a period of constant PET. As discussed above, ET is assumed to be equal to PET, until the water content in the entire root zone falls to  $\theta_{ll}$ . Then, there is a steep drop in ET to a value equal to the upward flux from the water table. Such abrupt changes are very rare in natural situations and better methods can be devised to handle the transition, as water is removed from the root zone. Actually, the rate that water can be removed from the root zone is a function of soil water potential (Figure 2-16).

The rate,  $E_r$ , that water can be removed from the root zone to satisfy ET demand could be calculated from a relationship such as the one developed by Norero (1969):

$$E_r = PET / (1 + (\psi/\psi^*)^k) \quad (2-25)$$

Where  $k$  is a constant that can be defined using methods given in Taylor and Ashcroft (1972) and Norero (1969),  $\psi$  is the soil water potential in the root zone which could be obtained from the soil water characteristic using the average root zone water content, and  $\psi^*$  is the value of  $\psi$  when  $E_r = 0.5$  PET. Inclusion of Equation 2-24 or a similar method in DRAINMOD would likely improve predictions for periods when the dry zone approaches the root zone depth. However, these modifications have not been made, nor tested at this time.

### Soil Water Distribution

The basic water balance equation for the soil profile (Equation 2-1) does not require knowledge of the distribution of the water within the profile. However, the methods used to evaluate the individual components, such as drainage and ET, depend on the position of the water table and the

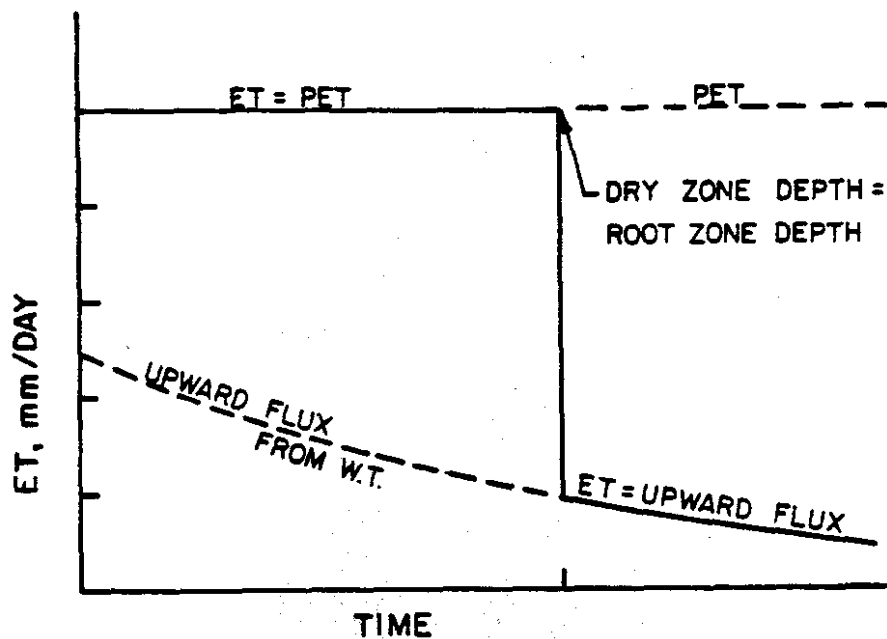


Figure 2-15. Schematic of the change in ET, with time for a constant PET as treated in the model. When the dry zone depth reaches the bottom of the root zone, ET is assumed to decline to the rate of upward flux.

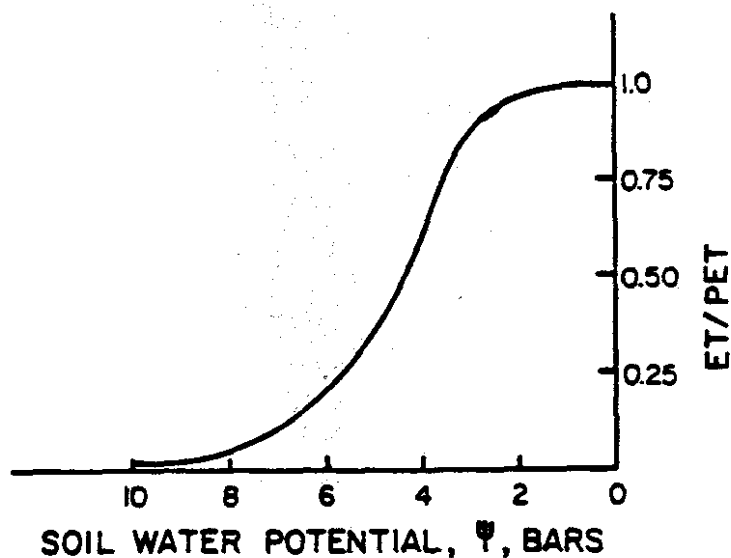


Figure 2-16. Schematic of relative evapotranspiration (ET/PET), as affected by soil water potential,  $\psi$ , in the root zone.

soil water distribution in the unsaturated zone. One of the key variables that is determined at the end of every water balance calculation in DRAINMOD is the water table depth. The soil water content below the water table is assumed to be essentially saturated; actually it is slightly less than the saturated value due to residual entrapped air in soils with fluctuating water tables. In some earlier models, the water content in the unsaturated zone was assumed to be constant and equal to the saturated value, less the drainable porosity. However, recent work (Skagges and Tang, 1976, 1978) has shown that, except for the region close to drains, the pressure head distribution above the water table during drainage may be assumed nearly hydrostatic for many field scale drainage systems. The soil water distribution under these conditions is the same as in a column of soil drained to equilibrium with a static water table. This is due to the fact that, in most cases in fields with artificial drains, the water table drawdown is slow and the unsaturated zone, in a sense, "keeps up" with the saturated zone. As a result, vertical hydraulic gradients are small. This is supported by the nearly vertical equipotential (H) lines in Figure 2-9 and Figure 2-17, which shows plots of pressure head versus depth at the drain, quarter and midpoints for drainage to open ditches spaced 20 m apart in a Panoche soil. The pressure head at the quarter and midpoints increase with depth in a 1:1 fashion indicating that the unsaturated zone is essentially drained to equilibrium with the water table (located where pressure head = 0), at all times after drainage begins.

The assumption of a hydrostatic condition above the water table during drainage will generally hold for conditions in which the D-F assumptions are valid. This will be true for situations where the ratio of the drain spacing to profile depth is large, but may cause errors for deep profiles, with narrow drain spacings.

Water is also removed from the profile by ET, which results in water table drawdown and changes in the water content of the unsaturated zone. In this case, the vertical hydraulic gradient in the unsaturated zone is in the upward direction. However, when the water table is near the surface, the vertical gradient will be small and the water content distribution still close to the equilibrium distribution. Solutions for the water content distribution in a vertical column of soil under simultaneous drainage and evaporation are given in Figures 2-18 and 2-19. The solutions to the Richards equation for saturated and unsaturated flow were obtained using numerical methods described by Skaggs (1974). The water table was initially at the surface of the soil column and solutions were obtained for various evaporation rates and a drainage rate at the bottom of the column equal to that resulting from drains spaced 30 m apart and 1 m deep.

The results in Figure 2-18 indicate that, when the water table is 0.4 m from the surface, the water content distribution for this soil is independent of evaporation rates less than 4.8 mm/day. When the rate of evaporation from the surface was 0.0, the water table fell to the 0.4 m depth after 1 day of drainage; whereas, it reached the same depth in 0.74 days, when the evaporation rate was 4.8 mm/day. However, the water content distribution above the water table was the same for both cases; it was also the same for the intermediate evaporation rate of 2.4 mm/day. Figure 2-19

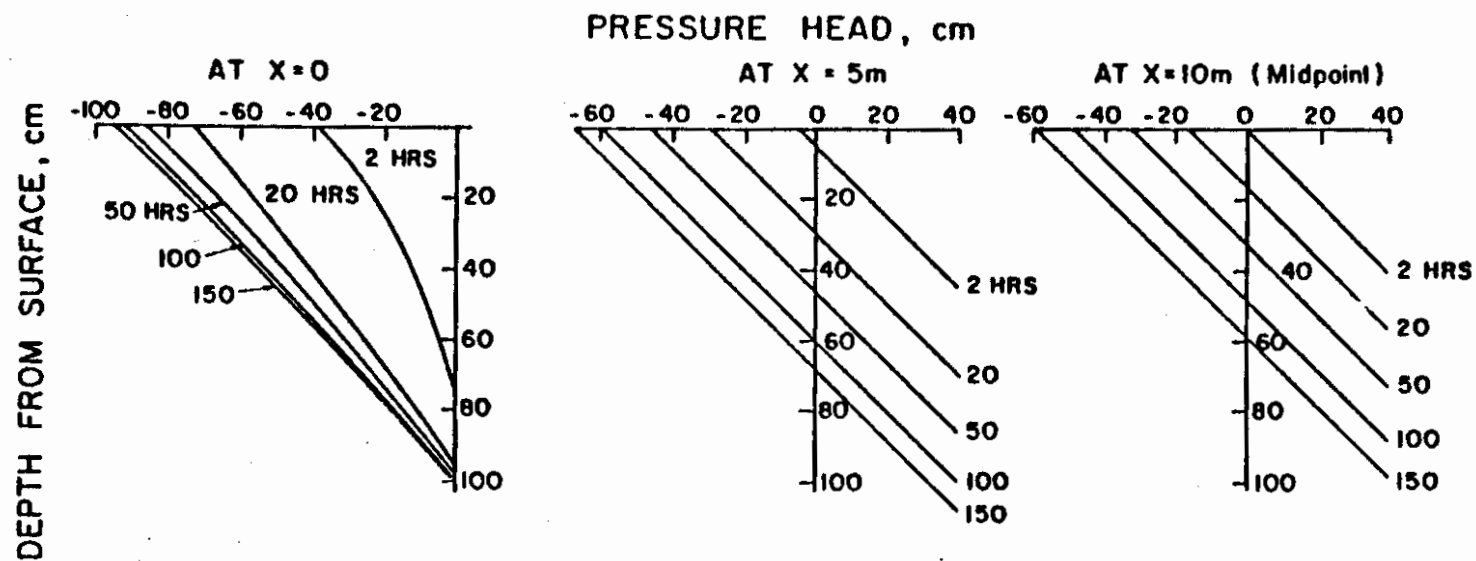


Figure 2-17. Pressure head distribution with depth at midpoint, quarter point and next to the drain for various times after drainage begins for a Panoche loam soil (after Skaggs and Tang, 1976).

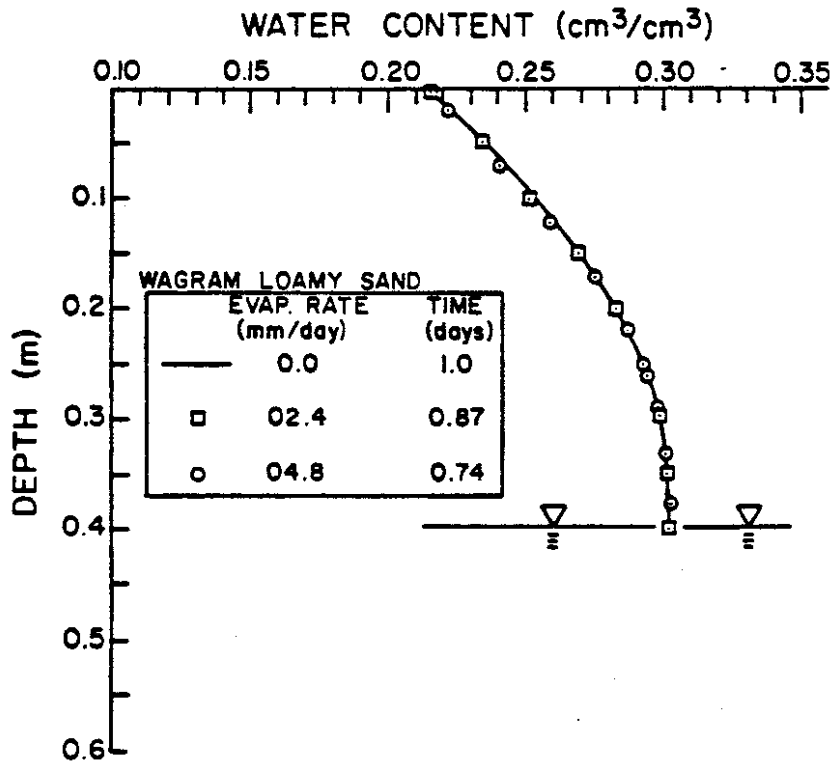


Figure 2-18. Soil water content distribution for a 0.4 m water table depth. The water table was initially at the surface and was drawn down by drainage and evaporation. Solutions are shown for three evaporation rates.

shows the distribution when the water table reached a depth of 0.7 m. Again, the soil water distribution was independent of the evaporation rate, except for the region close to the surface at the high evaporation rate (4.8 mm/day). The distribution for no evaporation is exactly the same as that which would result from the profile draining to equilibrium with a water table 0.7 m deep. Thus, the "drained to equilibrium" assumption, appears to provide a good approximation of the soil water distribution for this soil for both drainage and evaporation, when the water table depth is relatively shallow. Even when the water table is very deep, the soil water distribution for some distance above the water table will be approximately equal to the "equilibrium" distribution.

The zone directly above the water table is called the wet zone and the water content distribution is assumed to be independent of the means in which water was removed from the profile. Thus, the air volume or the volume of water leaving the profile by drainage, ET, and deep seepage, may be plotted as a function of water table depth as shown in Figure 2-20. Assuming hysteresis can be neglected, Figure 2-20 would allow the water

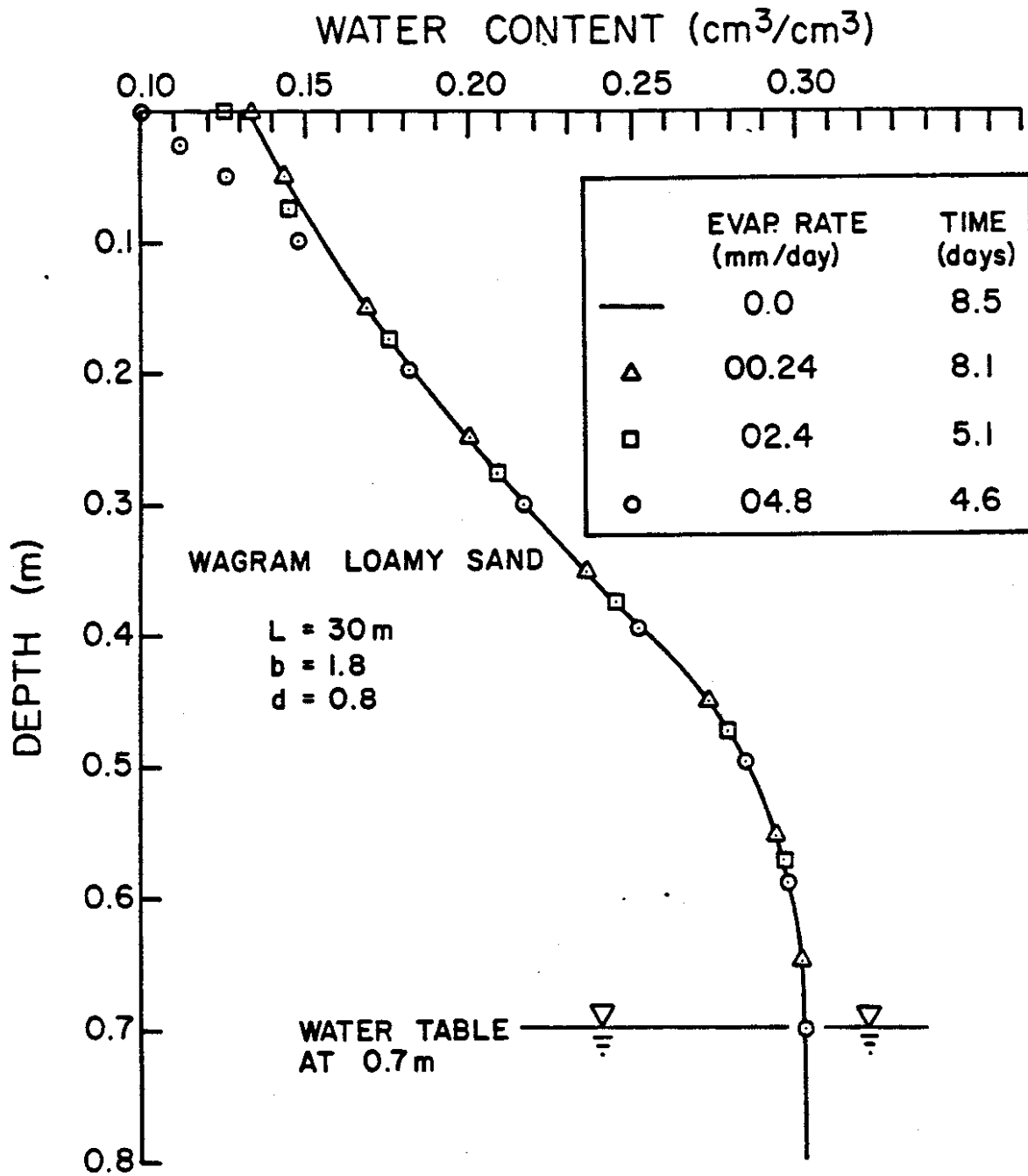


Figure 2-19. Soil water distribution for a water table depth of 0.7 m for various drainage and evaporation rates.

table depth to be determined simply from the volume of water that enters or is removed from the profile over an arbitrary period of time. For example, if the water table in the Wagram loamy sand of Figure 2-20 is initially at a depth of 0.6 m, the air volume above the water table would be  $V_a = 33$  mm. Then, if drainage and ET removed 10 mm of water during the following day, the total  $V_a$  will be 43 mm and the depth of the wet zone, which is equal to the water table depth in this case, 0.66 m (from Figure 2-20). Subsequent infiltration of 25 mm would reduce the air volume to 18 mm and the water table depth to 0.48 m.

The maximum water table depth for which the approximation of a drained to equilibrium water content distribution will hold depends on the hydraulic conductivity functions of the profile layers and the ET rate. The maximum depth will increase with the hydraulic conductivity of the soil and decrease with the ET rate. Because the unsaturated hydraulic conductivity decreases rapidly with water content, large upward gradients may develop near the surface, or near the bottom of the root zone, where the soil water

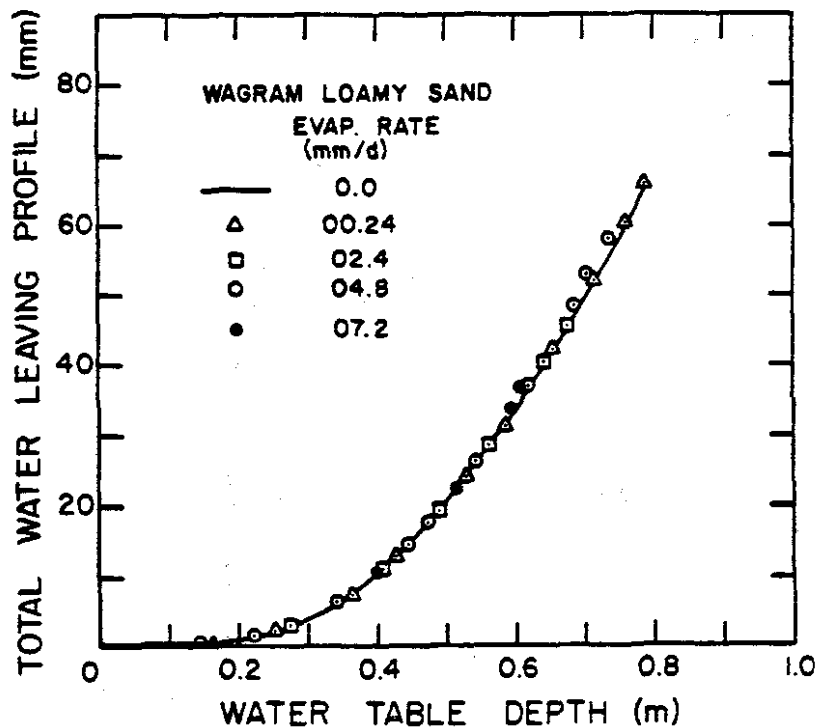


Figure 2-20. Volume of water leaving profile ( $\text{cm}^3/\text{cm}^2$ ) by drainage and evaporation versus water table depth. Solutions for five evaporation rates are given.

distribution departs from the equilibrium profile. At this point, the upward flux cannot be sustained for much deeper water table depths and additional water necessary to supply the ET demand would be extracted from storage in the root zone creating a dry zone as discussed in the ET section. This is shown schematically in Figure 2-21.

For purposes of calculation in DRAINMOD, the soil water is assumed to be distributed in two zones - a wet zone extending from the water table up to the root zone and possibly through the root zone to the surface, and a dry zone. The water content distribution in the wet zone is assumed to be that of a drained to equilibrium profile. When the maximum rate of upward water movement, determined as a function of the water table depth, is not sufficient to supply the ET demand, water is removed from root zone storage creating a dry zone as discussed in the ET section. The depth of the wet zone may continue to decrease due to drainage and some upward water movement. At the same time, the dry zone, with a constant water content of  $\theta_{LL}$  may continue to increase to a maximum depth equal to that of the root

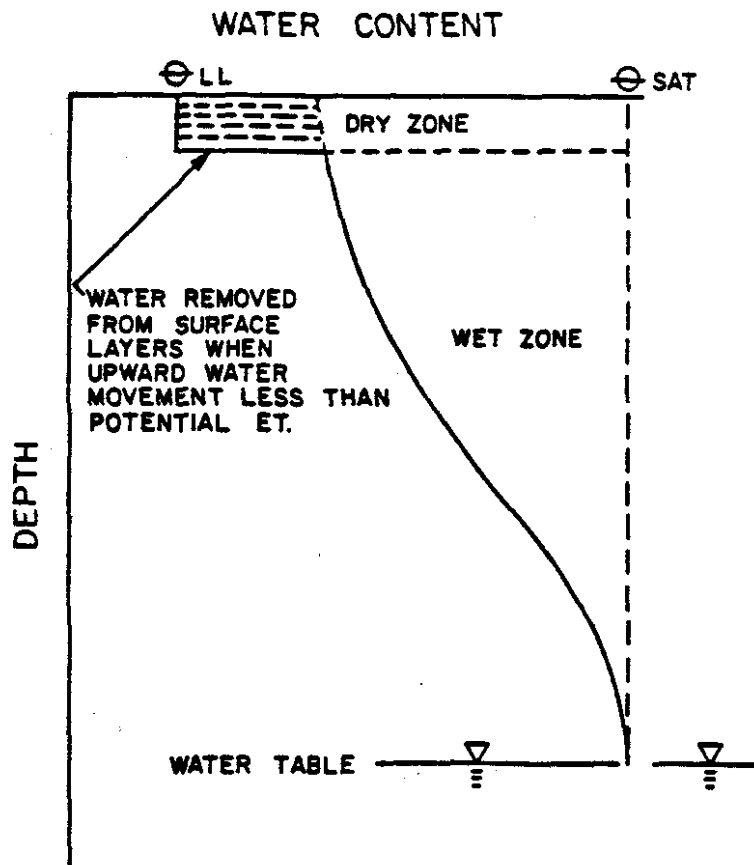


Figure 2-21. Schematic of soil water distribution when a dry zone is created near the surface.

zone. The water table depth is calculated as the sum of the depths of the wet and dry zones. When rainfall occurs, the storage volume in the dry zone, if one exists, is satisfied before any change in the wet zone is allowed. However, the depth to the water table will decrease by virtue of the reduction of the dry zone depth.

The assumptions made concerning soil water distribution may cause errors during periods of relatively dry conditions in soils with deep water tables and low K in the subsurface layers. Deep water tables may result from vertical seepage into an underlying aquifer or because of deep subsurface drains. For such conditions, the soil water at the top of the wet zone just beneath the root zone may be depleted by slow upward movement and by roots extending beyond the assumed depth of the concentrated root mass. Such conditions may cause the water content at the top of the wet zone to significantly depart from the drained to equilibrium distribution. However, this will not cause a problem for wet conditions and for most shallow water table soils for which the model was derived.

#### Rooting Depth

The effective rooting depth is used in the model to define the zone from which water can be removed as necessary to supply ET demands. Rooting depth is read into the model as a function of Julian date. Since the simulation process is usually continuous for several years, an effective depth is defined for all periods. When the soil is fallow, the effective depth is defined as the depth of the thin layer that will dry out at the surface. When a second crop or a cover crop is grown, its respective rooting depth function is also included. The rooting depth function is read in as a table of effective rooting depth versus Julian date. The rooting depth for days other than those listed in the table is obtained by interpolation.

This method of treating the rooting depth is at best an approximation. The depth and distribution of plant roots is affected by many factors, in addition to crop species and date of planting. These factors included barriers, fertilizer distribution, tillage treatments, and others, as reviewed in detail by Allmaras, et al, (1973) and Danielson (1967). A good discussion of the effect of various factors on root growth and distribution, with effective graphic presentations, is given in Chapter 1, Section 15 of the SCS-NEH. One of the most important factors influencing root growth and distribution is soil water. This includes both depth and fluctuation of the water table as well as the distribution of soil water during dry periods. Since the purpose of the model is to predict the water table position and soil water content, a model which includes the complex plant growth processes would be required to accurately characterize the change of the root zone with time. Such models have been developed for very specific situations, but their use is limited by input data and computational requirements at this time. Research is being conducted at North Carolina State University to develop root and plant growth models for use in DRAINMOD. Results of this and similar work at other locations should lead to future improvements in this component of the model.

The variation of root zone depths with time after planting may be approximated for some crops from experimental data reported in the literature. Studies of the depth and distribution of corn roots under field conditions were reported by Mengel and Barber (1974). Their data were collected on a silt loam soil which was drained, with drains placed 1 m deep and 20 m apart. They observed little evidence of root growth limitation by moisture or aeration stresses. The data of Mengel and Barber are plotted in Figure 2-22 for root zone depth versus time. Numbers on the curves indicate percentage of the total root length found at depths less than the value plotted. The broken sections of the curves were approximated by assuming that the effective root depth increases slowly for the first 20 days after planting, then more rapidly until the beginning of their measurements on day 30. The data of Mengel and Barber (1974) for the year 1971 showed the total root length reached a maximum 80 days after planting at about the silking stage, remained constant until day 94, then decreased until harvest at day 132. However, the percentage of roots less than a given depth remained relatively constant after about 80 days as shown in Figure 2-22.

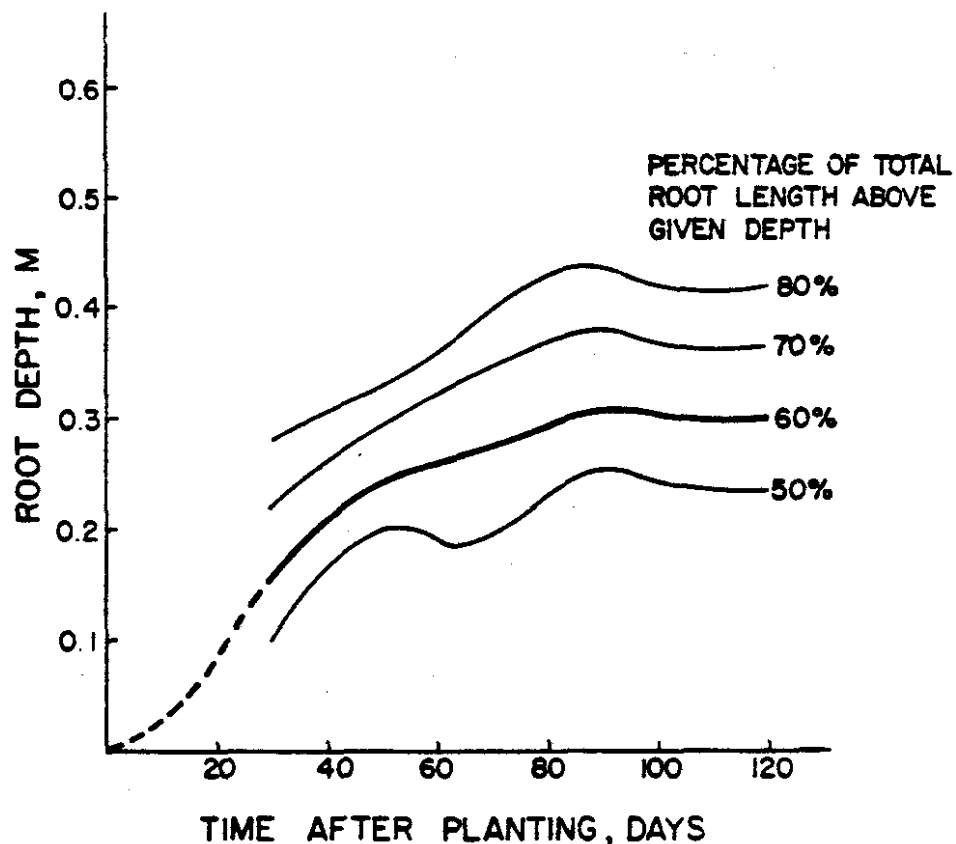


Figure 2-22. Relationships for depth above which 50, 60, 70, and 80 percent of the total root length exists versus time after planting for corn. From data given by Mengel and Barber (1974).

A similar study on the root distribution in corn was conducted by Foth (1962). Distribution plots based on root weights are given in Figure 2-23. The major differences between these results and those of Mengel and Barber were the shorter growing season (85 day versus 120 day corn) and smaller root depths, than those given in Figure 2-22. The total root dry weight is also plotted versus time in Figure 2-23. Foth found that root growth for plants less than 0.3 to 0.4 m reached a maximum by end of the vegetative growth stage 45 to 50 days after planting. After that date, there was a more rapid increase of roots, at deeper depths.

The following comments regarding moisture extraction patterns are made in the SCS-NEH (Section 16, Chapter 1, pages 1-30 and 1-33).

"For most plants, the concentration of absorbing roots is greatest in the upper part of the root zone (usually in the top foot) and near the base of the plant. Extraction of water is most rapid in the zone of greatest root concentration and under the most favorable conditions of temperature and aeration. Since water also evaporates from the upper few inches of soil, moisture is withdrawn rapidly from the upper part of the soil. As the amount of moisture in this part of the root zone is diminished, soil-moisture tension increases. Plants then get moisture from the lower parts of the root zone.

In uniform soils that are fully supplied with available moisture, plants use water rapidly from the upper part of the root zone and slowly from the extreme lower part. Basic moisture-extraction curves indicate that almost all plants growing in a uniform soil with an adequate supply of available moisture have similar moisture-extraction patterns. The usual extraction pattern shows that about 40 percent of the extracted moisture comes from the upper quarter of the root zone, 30 percent from the second quarter, 20 percent from the third quarter, and 10 percent from the bottom quarter. Values for individual crops are within a range of  $\pm 10$  percent.

It is apparent that input data to DRAINMOD for the effective rooting depth-time relationship should not be based on the maximum depth of root penetration. Use of the 60 percent curve, as shown by the dark curve in Figure 2-22 has given good results in tests of the model. Relationships such as those given in Figures 2-22 and 2-23 for corn are not available for many crops. Values for a constant effective root zone depth are reported in the literature for many crops and are used in irrigation design. Bloodworth, et al, (1958) reported root distribution data for several mature crops. Methods for estimating the effective root zone depth-time relationship from single effective depth values given in the literature are discussed in Chapter 5.

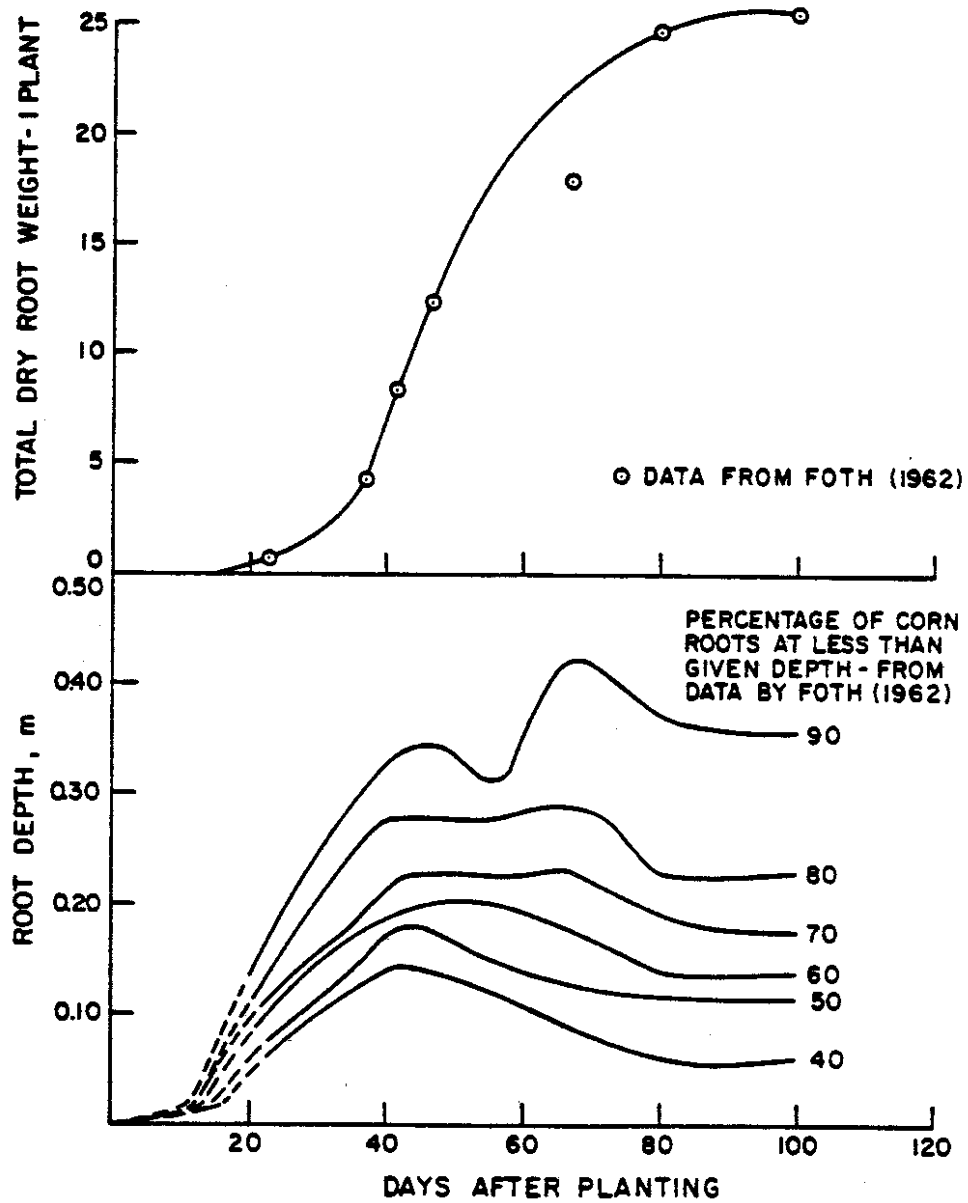


Figure 2-23. Root depths and total dry root weight versus times after planting for corn. From data given by Foth (1962).

