United States
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**Soil
Conservation
Service**

National Engineering Handbook

Section 3

Sedimentation

Chapter 4

Transmission of Sediment by Water

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Transport of Sediment by Water

Symbols

Symbol	Description	Unit
A	Area of flow, cross section	Feet
b	Channel width or water surface width	Feet
D	Depth of flow	Feet
d_{50}	Median size of sediment (letter d with numerical subscript denotes particle size in sediment for which the percentage by weight corresponding to subscript is finer, e.g., d_{84} is size for which 84 percent of sediment by weight is finer).	Millimeters, inches, or feet
d_m	Effective diameter of the bed material	Feet or millimeters
d_s	Particle size (unspecified)	Millimeters, inches, or feet
F_n or F_d	Froude number; equal to $\frac{U}{(gD)^{1/2}}$	Dimensionless
f	Darcy-Weisbach friction coefficient $\frac{8gRS}{U^2}$	Dimensionless
g	Acceleration due to gravity, 32.2	Feet per second per second *or*
	or 9.8	Meters per second per second
k_s	Representative grain size	Feet
Q	Water discharge	Cubic feet per second
Q_b	Bedload discharge	Tons per day or pounds per second
Q_s	Water discharge effective in transporting bedload	Cubic feet per second
Q_T	Total bed-material discharge	Tons per day or pounds per second
q	Unit water discharge	Cubic feet per second per foot of channel width
q_o or q_c	Unit water discharge just sufficient to move bed material	Cubic feet per second per foot channel width

q_B	Unit bedload discharge	Tons per day per foot or pounds per second per foot of channel width
q_T	Unit bed-material discharge	Tons per day per foot or pounds per second per foot of channel width
R	Hydraulic radius	Feet
R_b	Hydraulic radius with respect to the bed	Feet
R_N or R_e	Reynolds number; equal to $\frac{UD}{\nu}$ or $\frac{4UR}{\nu}$	Dimensionless
R_*	Boundary Reynolds number; equal to $\frac{U_* d_s}{\nu}$ (Shields)	Dimensionless
R'	Hydraulic radius with respect to the grain	Feet
R''	Hydraulic radius with respect to dunes and bars	Feet
S	Slope	Feet per foot
S_w	Water surface slope or hydraulic gradient	Feet per foot
S_o	Bed slope	Feet per foot
S_e	Energy gradient	Feet per foot
S_s	Specific gravity of sediment	Dimensionless
T°	Water temperature	Degrees Fahrenheit or degrees Celsius
u_*	Shear velocity $(gDS_e)^{1/2}$	Feet per second
u'_*	Shear velocity associated with grain roughness	Feet per second
U or V	Mean velocity	Feet per second
w	Fall velocity of sediment particles	Feet per second
γ	Unit weight of water, 62.4 *or* 1.0	Pounds per cubic foot *or* Grams per cubic centimeter
γ_s	Unit weight of sediment, dry	Pounds per cubic foot
Δ_γ	Difference between the specific weight of sediment and that of water	Pounds per cubic foot
δ	Thickness of laminar sublayer	Feet
θ	A form of the bed shear, τ_o	Dimensionless
ν	Kinematic viscosity	Square feet per second
μ	Dynamic viscosity	Pound-seconds per square foot
ρ	Density of water	Slugs per cubic foot
ρ_s	Density of sediment	Slugs per cubic foot
Ψ	A parameter indicating the ability of a flow to dislodge a given particle size (Einstein)	Dimensionless
Φ	A parameter describing the intensity of transport of bed material in a given size range (Einstein)	Dimensionless
τ_o	Total bed shear stress	Pounds per square foot
τ_c	Critical tractive stress associated with beginning of bed movement (Shields)	Pounds per square foot
τ'	Shear stress associated with grain resistance	Pounds per square foot
τ''	Shear stress associated with irregularities in bed and banks	Pounds per square foot

Antidunes. Bed forms that occur if the water velocity is higher than that forming dunes and plane beds. Antidunes commonly move upstream and are accompanied by and in phase with waves on the water surface.
Armor. A layer of particles, usually gravel size, that covers the bed as a coarse residue after erosion of the finer bed materials.
Bed form. Generic term used to describe a sand streambed. Includes ripples, dunes, plane bed, and antidunes (see fig. 4–3).
Bedload. Material moving on or near the streambed by rolling, sliding, and making brief excursions into the flow a few diameters above the bed.
Bed-material load. The part of the total load of a stream that is composed of particle sizes present in appreciable quantities in the shifting parts of the streambed.
Coefficient of viscosity. The ratio of shear stress to the velocity gradient perpendicular to the direction of flow of a Newtonian fluid or the ratio of shear stress in a moving liquid to the rate of deformation.
Coefficient of kinematic viscosity. The ratio of the coefficient of viscosity to the density of a fluid.
Dunes. Bed forms with a triangular profile having a gentle upstream slope. Dunes advance downstream as sediment moves up the upstream slope and is deposited on the steeper downstream slope. Dunes move downstream much more slowly than the stream flow.
Fall diameter or standard fall diameter. The diameter of a sphere that has a specific gravity of 2.65 and the same terminal velocity as a particle of any specific gravity when each is allowed to settle alone in quiescent distilled water of infinite extent and at a temperature of 24° C. A particle reaches terminal velocity when the water resistance is equal to the force of gravity.
Laminar flow. Low-velocity flow in which layers of fluid slip over contiguous layers without appreciable mixing.
Plane bed. A sedimentary bed with irregularities no larger than the maximum size of the bed material.
Ripples. Bed forms that have a triangular profile and are similar to dunes but much smaller.
Standing waves. Water waves that are in phase with antidunes.
Suspended load. The part of the total sediment load that moves above the bed layer. The weight of suspended particles is continuously supported by the fluid (see wash load).
Turbulent flow. A state of flow in which the fluid is agitated by crosscurrents and eddies.
Uniform flow. A flow in which the velocity is the same in both magnitude and direction from point to point along a reach.
Wash load. The part of the sediment load of a stream composed of fine particles (usually smaller than 0.062 mm) found only in relatively small quantities in the streambed. Almost all the wash load is carried in nearly permanent suspension, and its magnitude depends primarily on the amount of fine material available to the stream from sources other than the bed.

Understanding the principles of sediment transport by flowing water is essential to interpreting and solving many problems. The individual characteristics of water and sediment and their interaction directly affect the type and volume of material eroded and transported and the place and time of deposition. Evaluating channel instability, including erosion or aggradation, and predicting the performance of proposed channel improvements are problems that require knowledge of sediment transport and use of procedures pertaining to it. Information derived from following sediment-transport prediction procedures is used in determining requirements for storage of coarse sediment in debris basins and other types of structures.

This chapter includes a discussion of the characteristics of water as a medium for initiating the movement and transport of sediment. The reaction of material on the streambed to the hydraulic forces exerted and the effect of velocity and flow depth on the rate of bed-material transport are described. Formulas and procedures designed to predict the rate of bed-material transport are given and evaluated. Recommendations are made for applying these formulas and procedures to channel problems. The chapter concludes with a discussion of the mechanism of suspended-load transport and a description of a method for computing suspended-load yield from concentration and flow-duration data.

Factors Affecting Sediment Transport

The mechanism of entrainment and the rate at which sediment is transported depend on the characteristics of the transporting medium and on the properties and availability of particles.

Characteristics of Water as the Transporting Medium

The interrelated characteristics of water that govern its ability to entrain and move sedimentary particles are density, viscosity, and acidity.

Density is the ratio of mass to volume. Increasing the temperature of water increases its volume and decreases its density. With an increase in temperature from 40 to 100° C (104 to 212° F), water will expand to 1.04 times its original volume. In working with large volumes of moving water, the slight variations in density that result from temperature change are usually ignored.

Viscosity is the cohesive force between particles of a fluid that causes the fluid to resist a relative sliding motion of particles. Under ordinary pressure, viscosity varies only with temperature. A decrease in water temperature from 26.7 to 4.4° C (80 to 40° F) increases viscosity about 80 percent.

Changes in viscosity affect the fall velocity of suspended sediment and thereby its vertical distribution in turbulent flow (Colby and Scott 1965, p. 62). Increasing the viscosity lowers the fall velocity of particles, particularly very fine sands and silts.

A substantial decrease in water temperature and the consequent increase in viscosity smooth the bed configuration, lower the Manning "n" roughness coefficient, and increase the velocity over a sand bed (U.S. Department of the Army 1968).

The pH value is the negative logarithm (base 10) of the hydrogen-ion concentration. Neutral water has a pH value of 7.0. Acid water has a pH value lower than 7.0; alkaline water has a pH value higher than 7.0.

In acid waters sediment deposition may be promoted by the formation of colloidal masses of very fine sediments (flocculation) that settle faster than their component fine particles.

Laminar Sublayer

In turbulent flow, a thin layer forms adjacent to the bed in which the flow is laminar because the fluid particles in contact with the bed do not move. This is the laminar sublayer; the higher the velocity or the lower the viscosity, the thinner the sublayer. If the boundary is rough enough, its irregularities may project into the theoretical laminar sublayer, thereby preventing its actual development.

Although laminar flow is primarily related to fluid viscosity, turbulent flow is affected by a number of factors. In laminar flow, filaments of water follow parallel paths, but in turbulent flow, the paths of particles crisscross and touch, mixing the liquid. A criterion defining the transition from laminar to turbulent flow is the Reynolds number, R_e—a ratio of inertial force to shear force on the fluid particle. If the Reynolds number is low, shear forces are dominant, but as the Reynolds number increases, they decline to little significance, thereby indicating the dominance of inertial forces.

The association of laminar flow with viscosity and that of turbulent flow with inertia are the same whether the fluid is moving or at rest. A small particle of sediment, such as very fine sand, settling in still or flowing water moves slowly enough to sustain laminar flow lines in relatively viscous media. Inertial forces become increasingly important as grain size increases and are dominant when the particle size exceeds 0.5 mm.

Characteristics of Transportable Materials

The characteristics of discrete particles are discussed in Chapter 2. The entrainment and transport of granular materials depend on the size, shape, and specific weight of the particles and their position with respect to each other. The resistance of cohesive materials depends largely on the forces of interparticle bonding. Cohesive forces can be attributed to several factors, including the amount and kind of clay minerals, the degree of consolidation or cementation, and the structure of the soil mass.

Forces Acting on Discrete Particles

Turbulence is a highly irregular motion characterized by the presence of eddies. The degree to which eddies form depends on the boundary roughness and geometry of the channel, and eddies are sustained by energy supplied by the flow. The eddies penetrate the laminar sublayer formed along the bed. Discrete particles resting on the bed are acted on by two components of the forces associated with the flow. One component force is exerted parallel to the flow (drag force) and the other is perpendicular to the flow (lifting force). Drag force results from the difference in pressure between the front and the back sides of a particle. Lifting force results from the difference in pressure on the upper and lower surfaces. If the lifting force exceeds the particle's immersed weight and the interference of neighboring grains, the particle goes into suspension.

Because turbulence is random and irregular, discrete particles tend to move in a series of short, intermittent bursts. In each burst, particles move a short distance and many grains move simultaneously. Then the movement subsides until another burst occurs. The frequency and extent of movement increases with the intensity of turbulence, and above a certain intensity some particles may be projected into the flow as suspended load (Sutherland 1967). The coarser and rounder the particles, however, the greater the possibility that they will begin to roll and continue rolling.

Tractive Force

Experiments to determine the forces that act on particles on a streambed were performed mainly to predict channel stability. More advanced methods are necessary to describe transport.

The instantaneous interactions between turbulent flow and discrete sediment particles resting on the bed were described briefly in the preceding paragraphs. In practical application, however, it is more convenient to deal with time-average values of the force field generated by the flow near the bed. Here, the forces normal to the bed having a time average equal to zero can be eliminated and only those forces parallel to the bed need to be considered. The time average of these forces is the tractive force. The tractive force measured over a unit surface area is the tractive stress. In a prismatic channel reach of uniform flow bounded by two end sections, the mean value of tractive stress is equal to the weight of the water prism in the reach multiplied by the energy gradient and divided by the wetted boundary surface in the reach. Shear stress or force per unit area of bed is expressed as $\tau_o = \gamma R S_e$.

Determining Critical Tractive Stress

The most widely used and most reliable evaluation of tractive stress related to the initiation of motion is that developed by Shields (1936). The theoretical concepts, supported by experiments, resulted in a plot of $\frac{\tau_c}{\gamma(\frac{\gamma_s}{\gamma} - 1)d_s}$ against $\frac{U_* d_s}{\nu}$. The first expression is an entrainment function and the second is the boundary Reynolds number, indicating the intensity of flow turbulence around the particle. The Shields data are based on particles of uniform size and a flat bed. The Shields experiments indicate that beyond a certain value of the boundary Reynolds number, $\frac{U_* d_s}{\nu}$, the value of the parameter $\frac{\tau_c}{\gamma(\frac{\gamma_s}{\gamma} - 1)d_s}$ remains constant. Within these limits, the critical tractive stress is therefore proportional to grain size.

Data on critical tractive stresses obtained in a number of investigations were assembled by Lane (1955). These data show that the critical tractive stress in pounds per square foot is equal to $\tau_c = 0.5\ d_{75}$, where d_{75} is the size in inches of the bank material at which 25 percent by weight is larger. The limiting (allowable) tractive stress was determined from observations of canals (Lane 1955). The recommended limiting tractive stress in pounds per square foot is equal to 0.4 of the d_{75} size in inches for particles that exceed 0.25 in diameter. Results of experiments on finer particles vary considerably, probably because of variations in experimental conditions. These include differences in interpreting the initiation of sediment movement, in temperature of the water, in concentration of colloids, and in configuration of the bed. Critical conditions for initiating movement sometimes are determined by the number of particles or the frequency with which the particles start to move. For example, one observer's criterion is the time at which particles begin to move every 2 seconds at a given spot on the bed (Sutherland 1967).

In figure 4–1a the critical tractive (shear) stress is plotted against the mean particle size or to the d_{75}. The figure shows the differences in critical tractive stress resulting from temperature variation and the boundary Reynolds number at various tractive stress levels. The wide departure of Lane's curve for critical tractive stress from the others in figure 4–1a is believed to be due to Lane's use of the data of Fortier and Scobey (1926) from canals after aging. The stability of some soils is increased by aging.

Determining Critical Velocity

Determining the critical velocity (the velocity at which particles in the bed begin to move) is another method for establishing stability criteria. Figure 4–1b shows critical water velocity as a function of mean grain size. There has been less agreement on critical velocity than on critical tractive stress, probably because bottom velocity increases more slowly with increasing depth than does mean velocity. Critical conditions for initiating movement can be expressed directly in terms of tractive stress, but critical mean velocity must be related to variation in velocity with depth.

Determining the correct critical value for tractive stress or velocity is important when considering stability problems in channels in which there is to be no significant movement of the boundary material. The significance of the critical value is determined by the magnitude and duration of flows that initiate sediment movement. A prolonged flow slightly exceeding the critical value may have little significance in terms of the volume of bed material transported. On the other hand, a brief flow substantially exceeding the critical value could transport a large volume of sediment.

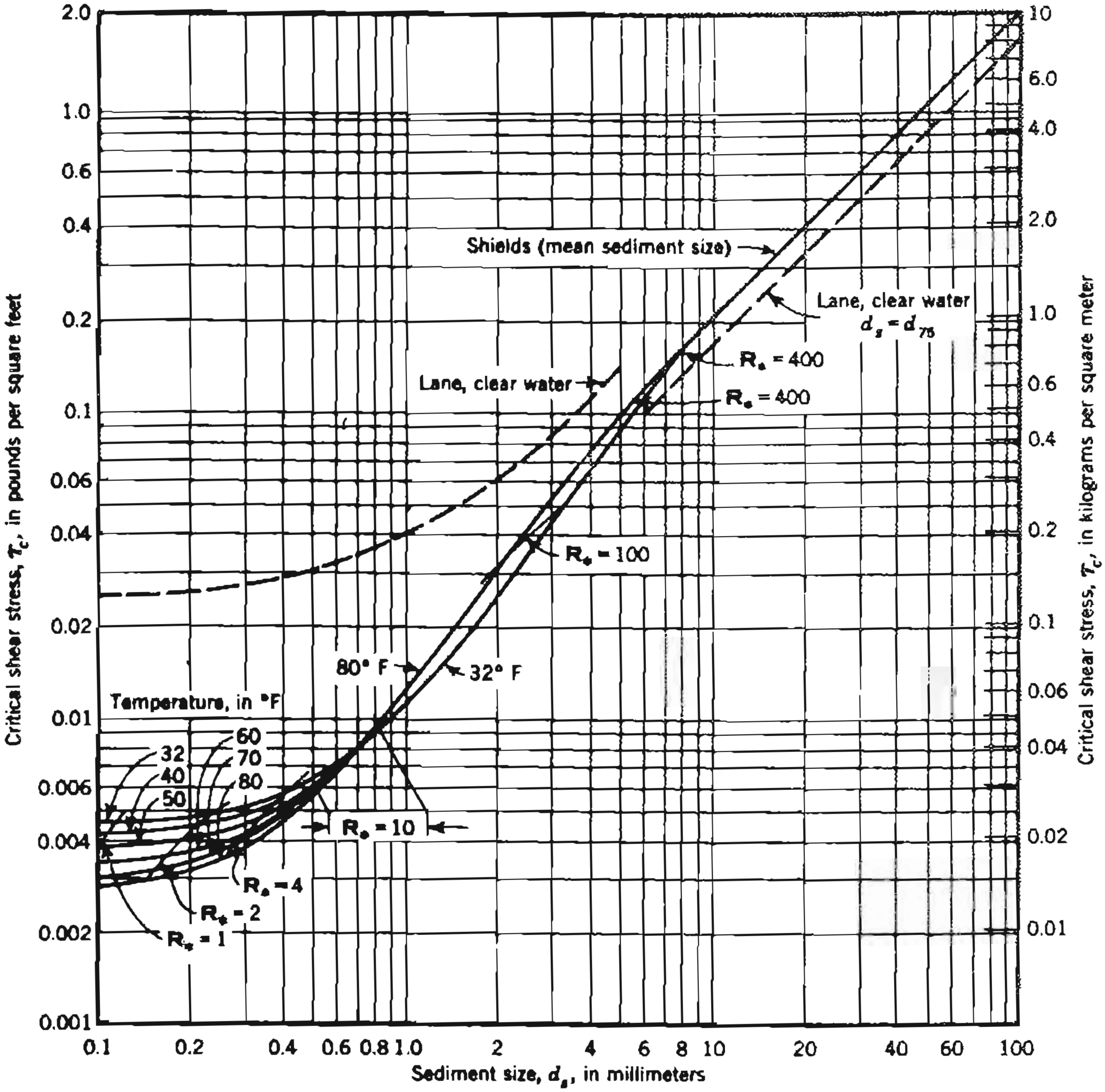

Figure 4–1a.—Critical shear stress for quartz sediment in water as a function of grain size. From Shields (1936), Lane (1955), and American Society of Civil Engineers (1975, p. 99).

Fixed Boundaries

The relationships of velocity, stage, and discharge for stream channels with fixed boundaries have long been satisfactorily predicted by selecting the appropriate "n" value in Manning's and other related formulas.

Movable Boundaries

Study of the hydraulics of movable boundaries has been directed to two general problems. Primary interest has been in determining methods for predicting the friction coefficient and thereby the correct velocity, stage, and discharge relationships for channel design. The need for these data as a key element in predicting sediment transport has added incentive to the investigations. The changes in bed form produced on a movable bed and the consequent change in friction characteristics of the bed have therefore become one of the most intensively studied flow phenomena. The literature on this subject generally describes the sequence of changes in bed configuration that can occur as the flow and transport intensity increase.

Ripples, ripples on dunes, or dunes may form at a low transport rate, and antidunes or a flat bed may form at a high transport rate. These bed forms have been observed in sand-bed flumes and streams with a d_{50} size finer than 1.0 mm. The variety of bed forms in coarser material seems to be smaller.

Pioneering efforts in investigating the hydraulics of movable beds led to dividing the hydraulic radius into two parts. One part is the radius resulting from the roughness of the grain size of the individual particles (R′), and the other is the radius resulting from the roughness of the bed configuration (R″) (Einstein 1950; Einstein and Barbarossa 1952).

From field observations Einstein and Barbarossa developed a graph relating the dimensionless ratio $\frac{V}{U''_*}$ (where $U''_* = (gR''S_e)^{1/2}$) to Einstein's flow-intensity parameter, Ψ. This graph indicates that for a given set of conditions it is possible to develop a unique stage-discharge relationship and thus to predict the hydraulics of a channel with movable boundaries. Vanoni and Brooks (1957) presented a graphical solution to the friction equation from which R′ is determined.

Another procedure for predicting hydraulic behavior in movable channel beds is based on the division of slope, S, into two parts, S′ and S″ (Meyer-Peter and Muller 1948). In this procedure S′ is the energy gradient associated with the grain size of the bed material under a certain velocity and depth, excluding form resistance, and S″ is the additional gradient pertaining to bed-form resistance. This division of slope was adopted by Alam and Kennedy (1969), whose procedure is explained in the appendix to this chapter.

A similar hydraulic consideration sometimes used as part of the preliminary procedure in sediment

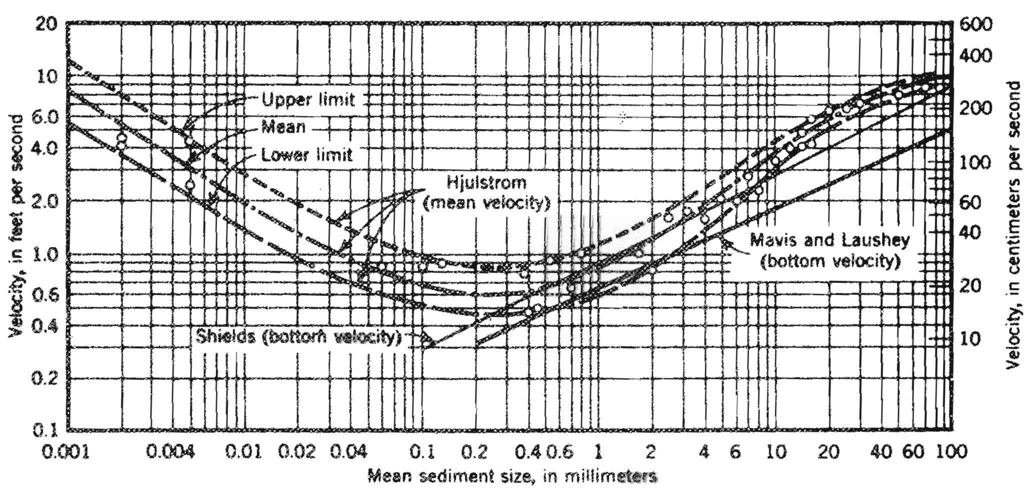

Figure 4–1b.--Critical water velocity for quartz sediment as a function of mean grain size. From American Society of Civil Engineers (1975, p. 102).

transport computations is the treatment of bank friction as completely distinct from bed friction. One such approach, involving use of Manning's friction equation, is included as part of the procedure in the Einstein bedload function.

In this discussion the term "bed-material load" is defined as that part of the total sediment load (suspended load plus bedload) that is composed of grain sizes occurring in appreciable quantities in the bed material. The part of the total load that consists of grain sizes not present in the bed material in significant quantities is the wash load. Sand-size particles that constitute all or the major part of the bed material travel either on the bed as bedload or in suspension. Figure 4–2 illustrates how the total sediment load is classified—bedload, bed-material load, and wash load. Evaluation techniques are not refined enough to predict accurately what part of the bed-material load moves in suspension or what part moves as bedload under specific hydraulic conditions. Establishing this separation does not seem essential to the general solution of sediment transport problems.

Transport rates for sand and gravel have been determined by both direct measurement and computation. Measurements of the transport rate in natural streams have been few, chiefly because of the difficulty in getting representative measurements. Sampling equipment established in or on the bed tends to alter the direction of flow filaments and the sediment concentration. The more accurate measurements have been made by using equipment installed to withdraw representative samples of the water-sediment mixture during specific periods. Another method is to sample the total load as the flow moves over a sill at an elevation the same as that of the slope upstream.

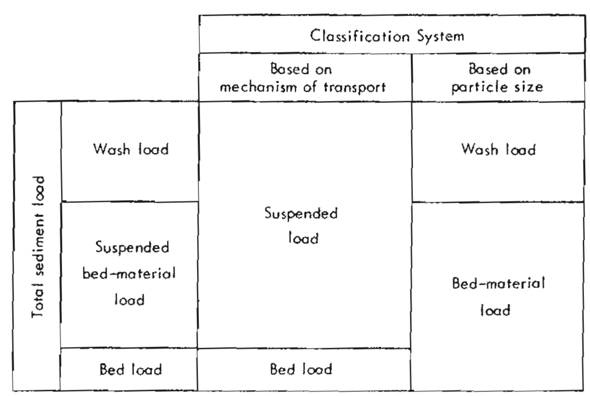

Figure 4–2.—Sediment load classification. Adapted from Cooper and Peterson (1970, p. 1,881).

The existence of many procedures for predicting transport rates indicates both the difficulty of obtaining measurements and the influence of many variables on the consistency of results. Because flume studies are the most easily controlled and exclude some variables, they have become the primary means of establishing relationships between stream discharge and bed-material load.

The earliest bed-material transport formula still in use is that of Duboys, who published results of studies of the Rhone River in 1879. Duboys originated a concept common to many later formulas when he assumed in his derivation that the rate of sediment transport is proportional to the tractive stress in excess of the critical value required to initiate motion.

The Duboys formula is

$$q_T = \Psi \tau_o (\tau_o - \tau_c) \tag{4-1}$$

where

q_T = rate of sediment transport per unit width of stream;

Ψ = a coefficient that depends on characteristics of the sediment (not to be confused with Einstein's Ψ);

τ_c = a value established by experiment (not the same as that of Shields).

Early in the twentieth century, several flume studies of sand transport were started, including that of Shields. He is best known for developing criteria for the initiation of movement. Probably the most extensive early investigation of sediment transport in flumes was Gilbert's in about 1910 (Gilbert 1914). Descriptions of a number of transport phenomena resulted from those experiments, but no general formula was derived.

Of the formulas that follow, those of Schoklitsch, Meyer-Peter, Haywood, and Meyer-Peter and Muller are bedload formulas. The Einstein bedload function, the Engelund-Hansen procedure, and the Colby procedure determine the rate of bed-material transport, both bedload and suspension load.

Schoklitsch Formula

Schoklitsch developed one of the more extensively used empirical formulas (Shulits 1935; Shulits and Hill 1968). He used his own experimental data and also data from Gilbert's flume measurements.

The 1934 Schoklitsch formula in English units is

$$q_B = \frac{86.7}{(d_{50})^{1/2}} S_e^{3/2} (q - q_o) \tag{4-2}$$

where

q_B = unit bedload discharge (pounds per second per foot of width);

d_{50} = medium size of sediment (inches);

$q_o = 0.00532 \dfrac{d_{50}}{S_o^{\ 4/3}}$.

In describing the formula, Shulits recommended using a cross section in a straight reach of river where the depth of water is as uniform as possible and the width changes as little as possible with stage. As described by Shulits, the Schoklitsch formula fits Gilbert's measurements for uniform particle sizes of about 0.3 to 7 mm and slopes ranging from 0.006 to 0.030 ft/ft for small particles and 0.004 to 0.028 ft/ft for larger particles.

Meyer-Peter Formula

In 1934 the Laboratory for Hydraulic Research at Zurich, Switzerland, published a bedload transport formula based on flume experiments with material of uniform grain size. The original analysis of the Zurich and Gilbert data for uniform particles ranging from about 3 to 28 mm in diameter was supplemented by studies of mixtures of various-sized particles up to 10 mm and having various specific gravities.

The Meyer-Peter formula in English units is

$$q_B = (39.25\, q^{2/3} S_o - 9.95\, d_m)^{3/2} \tag{4-3}$$

where d_m is expressed in feet. The new term in this formula is d_m, the effective diameter of the bed material, which identifies the characteristic size of a sample. To determine this value, divide the size distribution curve of a bed-material mechanical analysis into at least 10 equal size fractions and determine the mean size and weight percentage of each fraction.

Haywood Formula

The Haywood formula is based on Gilbert's flume data and data from the U.S. Waterways Experiment Station, Vicksburg, Miss. In his evaluation, Haywood (1940) adjusted Gilbert's data to account for sidewall resistance. He assumed that the discharge effective in moving bedload is midway between the discharge of walls offering no resistance and that of walls offering the same resistance as the bed. Haywood demonstrated the close relationship of his formula to the Schoklitsch formula, which is based on some of the same data. Haywood believed that his formula substantially agrees with Scholkitsch's formula for relatively large rates of bedload movement and that it is much more accurate for very small rates of movement. Haywood considered 3 mm to be the maximum particle size for application of his formula. He regarded his formula as a modification of the Meyer-Peter formula.

The Haywood formula is

$$q_B = \left(\frac{q^{2/3} S_o - 1.20\, d^{4/3}}{0.117\, d^{1/3}} \right)^{3/2} \qquad \textbf{(4–4)}$$

where d is d_{35} expressed in feet.

Meyer-Peter and Muller Formula

The Meyer-Peter and Muller formula is based on data obtained from continuing the experiments that resulted in the Meyer-Peter formula. The range of variables, particularly slope, was extended. A few tests were run with slopes as steep as 20 percent and sediment sizes as coarse as 30 mm. Meyer-Peter and Muller stated explicitly that their work was on bedload transport, by which they meant the movement of sediment that rolls or jumps along the bed. Transport of material in suspension is not included (Meyer-Peter and Muller 1948).

The Meyer-Peter and Muller formula as translated by Sheppard (1960) is

$$q_B = 1.606 \left[3.306 \quad \frac{Q_S}{Q} \; \frac{{d_{90}}^{1/6\ 3/2}}{n_S} DS_e - 0.627\, d_m \right]^{3/2} \qquad \textbf{(4–5)}$$

where d_{90} and d_m are expressed in millimeters.

Nomographs are available for determining $\frac{Q_s}{Q}$ (a ratio of the discharge quantity determining bedload transport to the total discharge) and n_S (a Manning "n" value for the streambed). The formula, a significant departure from the previously cited formulas, includes a ratio of the form roughness of the bed to the grain roughness of the bed surface.

Einstein Bedload Function

In 1950 Einstein's bedload function had a major effect on investigations of the hydraulics and sediment transport characteristics of alluvial streams. Einstein (1950) described the function as "giving rates at which flows of any magnitude in a given channel will transport as bed load the individual sediment sizes of which the channel bed is composed." It was developed on the basis of experimental data, theory of turbulent flow, field data, and intuitive concepts of sediment transport.

The Einstein bedload function first computes bedload and then, by integrating the concentration at the bed layer with the normal reflection of that concentration in the remainder of the flow depth, determines the total bed-material load.

Einstein introduced several new ideas into the theory of sediment transport. Included were new methods of accounting for bed friction by dividing it into two parts, one pertaining to the sand-grain surface and the other to the bed-form roughness, such as ripples or dunes. An additional friction factor, that of the banks, is included in the procedure for determining hydraulic behavior before computing bed-material transport.

Another idea introduced by Einstein to explain the bedload function is that the statistical properties of turbulence govern the transport of particles as bedload. This statistical character is reflected in the structure of the dimensionless parameter ϕ, defined as the intensity of bedload transport. The relationship between this factor and the dimensionless flow intensity, Ψ (another dimensionless parameter reflecting the intensity of shear on the particle) is used in the procedure. The ϕ-Ψ relationship has subsequently been tested by others and found to be an appropriate determinant of bedload transport.

Engelund-Hansen Procedure

Engelund and Hansen (1967) developed a procedure for predicting stage-discharge relationships and sediment transport in alluvial streams. They introduced a parameter θ (the reciprocal of Einstein's Ψ) to represent the ratio of agitating forces (horizontal drag and lifting force) to the stabilizing force (immersed weight of the particle). This parameter is a dimensionless form of the bed shear, τ_0, to be divided into two parts: τ', the part acting directly as traction on the particle surface, and τ'', the residual part corresponding to bed-form drag. This division is similar to that of the Einstein-Barbarossa R′ and R″. The authors' diagram of the relationship of bed forms to the two separations of total bed shear and to velocity is shown in figure 4–3. Principles of hydraulic similarity were used to develop a working hypothesis for describing total resistance to flow, specifically for dune-covered streambeds and bed-material discharge.

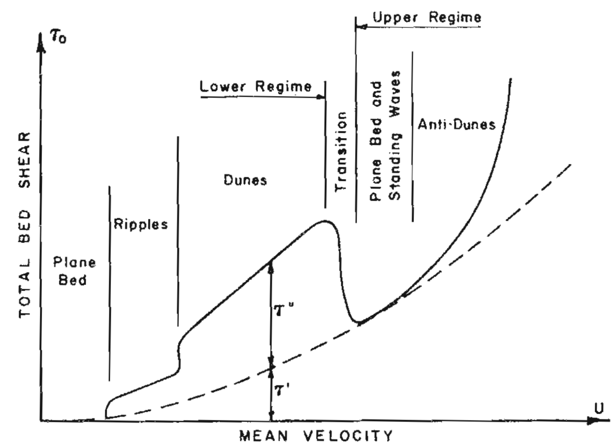

Figure 4–3.—Relationship between grain roughness (τ') and form drag (τ'') and total bed shear (τ_0). From Engelund and Hansen (1967).

The steps used in applying the Engelund-Hansen procedure are given here in some detail because the procedure demonstrates the impact of changing bed forms on bed-material transport and because it was published in a foreign journal not readily available for reference. Data from flume experiments by Guy and by Simons and Richardson (Guy, Simons, and Richardson 1966) were used to test the Engelund-Hansen theories. The mean sizes used in these experiments were 0.19, 0.27, 0.45, and 0.93 mm. Transport of the bed material, both in suspension and moving along the bed, was measured.

The Engelund-Hansen procedure includes both a simplified and a more detailed series of computations. Figure 4–4 in conjunction with figure 4–3 shows the flow regime in which a semigraphical solution, figure 4–5, applies; that is, in the region of dune formation.

The steps in applying the graphical form are as follows:

Example 1 (using the authors' symbols)

Given:

D = 1.219 m
d = mean fall diameter = 3.2×10^{-4} m
S_O = slope of the channel = 2.17×10^{-4}
S_S = specific gravity of sediment = 2.68

Calculate the ratio of the mean depth, D, to the mean fall diameter, d, of the bed material.

$$\frac{D}{d} = \frac{1.219}{3.2 \times 10^{-4}} = 3.81 \times 10^3$$

where

$$S_O \text{ (fig. 4–5)} = 2.17 \times 10^{-4}$$

$$\left[\frac{q}{(S_S - 1)gd^3}\right]^{1/2} = 3.3 \times 10^4 \text{ and } \Phi = 1.5$$

then

$$\begin{aligned} q &= [(S_S - 1)gd^3]^{1/2}(3.3 \times 10^4) \\ &= [1.68(9.8)(3.2 \times 10^{-4})^3]^{1/2}(3.3 \times 10^4) \\ &= 0.766 \text{ m}^3/(\text{s}\cdot\text{m}) = 8.25 \text{ ft}^3/(\text{s}\cdot\text{ft}) \end{aligned}$$

and

$$\begin{aligned} q_T &= \Phi[(S_S - 1)gd^3]^{1/2} \\ &= 1.5[1.68(9.8)(3.2 \times 10^{-4})^3]^{1/2} \\ &= 3.48 \times 10^{-5}\text{m}^3/\text{s}\cdot\text{m}) \\ &= 0.000375 \text{ ft}^3/(\text{s}\cdot\text{ft}) \end{aligned}$$

At 95 lb/ft³, sediment by weight is 95 × 0.000375 = 0.036 lb/(s·ft)

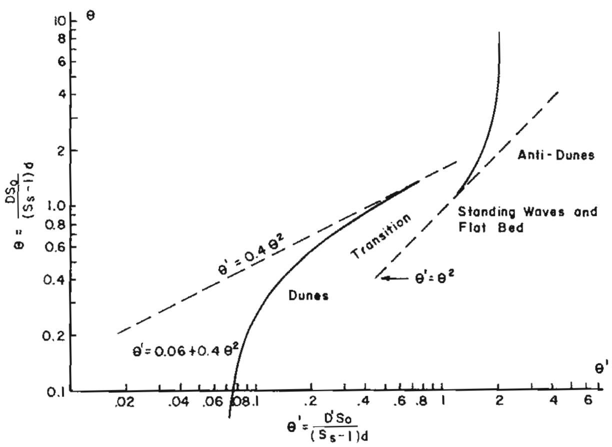

Figure 4–4.—Relationship between dimensionless forms of bed shear (θ and θ'). From Engelund and Hansen (1967) and American Society of Civil Engineers (1975, p. 135).

Example 2 shows early in the computation that the long form of computations must be followed.

Given:

D = mean depth of 1.0 ft = 0.3048 m
d = mean fall diameter of 3.2×10^{-4} m
S_s = 2.68
S_o = slope of the channel = 0.002

$$\frac{D}{d} = \frac{0.3048}{3.2 \times 10^{-4}} = 9.52 \times 10^2$$

These values fall to the right of the lined chart (fig. 4–5) and probably within the transition and plane-bed regime.

$$\theta \text{ (see figs. 4–3 and 4–4} = \frac{DS_o}{(S_s - 1)d} \quad \frac{(0.3048)(0.002)}{(1.68)(0.00032)}$$

$$= 1.134$$

where

θ' = for transition or plane bed regime
$= 0.4\,\theta^2 = 0.514$

D' = boundary layer thickness =

$$\frac{\theta' D}{\theta} = \frac{0.514}{1.134}\ (0.3048)$$

$$= 0.138 \text{ m}$$

k = surface roughness as determined by Engelund-Hansen

$= 2.5\ d = 2.5(0.32) = 0.80$ mm

$$\frac{U}{[gD'S_o]^{1/2}} = 6.0 + 5.75 \log \frac{D'}{k} \text{ in millimeters}$$

$$U = [9.8(0.138)(0.002)]^{1/2}$$

$$\left[6.0 + 5.75 \log \frac{138}{0.80}\right] = 0.98 \text{ m/s}$$

$$U = 3.22 \text{ ft/s}$$

$$\therefore \text{discharge} = 3.22 \text{ ft}^3/(\text{s}\cdot\text{ft})$$

The bed-material discharge can be calculated as follows:

$f\Phi = 0.1\,\theta^{5/2}$
(as determined by Engelund-Hansen)

where

$$f = \text{friction factor} = \frac{2g\,S_oD}{U^2}$$

$$= \frac{2(9.8)(0.002)(0.3048)}{(0.981)^2} = 0.0124$$

then

$$\Phi = \frac{0.1}{f}\,\theta^{5/2} = \frac{0.1}{0.0124}\ 1.134^{5/2} = 11.04$$

and

$$q_T = \Phi[(S_s - 1)gd^3]^{1/2}$$
$$= 11.04[1.68 \times 9.8(3.2 \times 10^{-4})^3]^{1/2}$$
$$= 2.564 \times 10^{-4} \text{m}^3/(\text{s}\cdot\text{m}) = 2.76 \times 10^{-3} \text{ ft}/(\text{s}\cdot\text{ft})$$

At 95 lb/ft^3, sediment by weight is 0.262 lb/(s·ft).

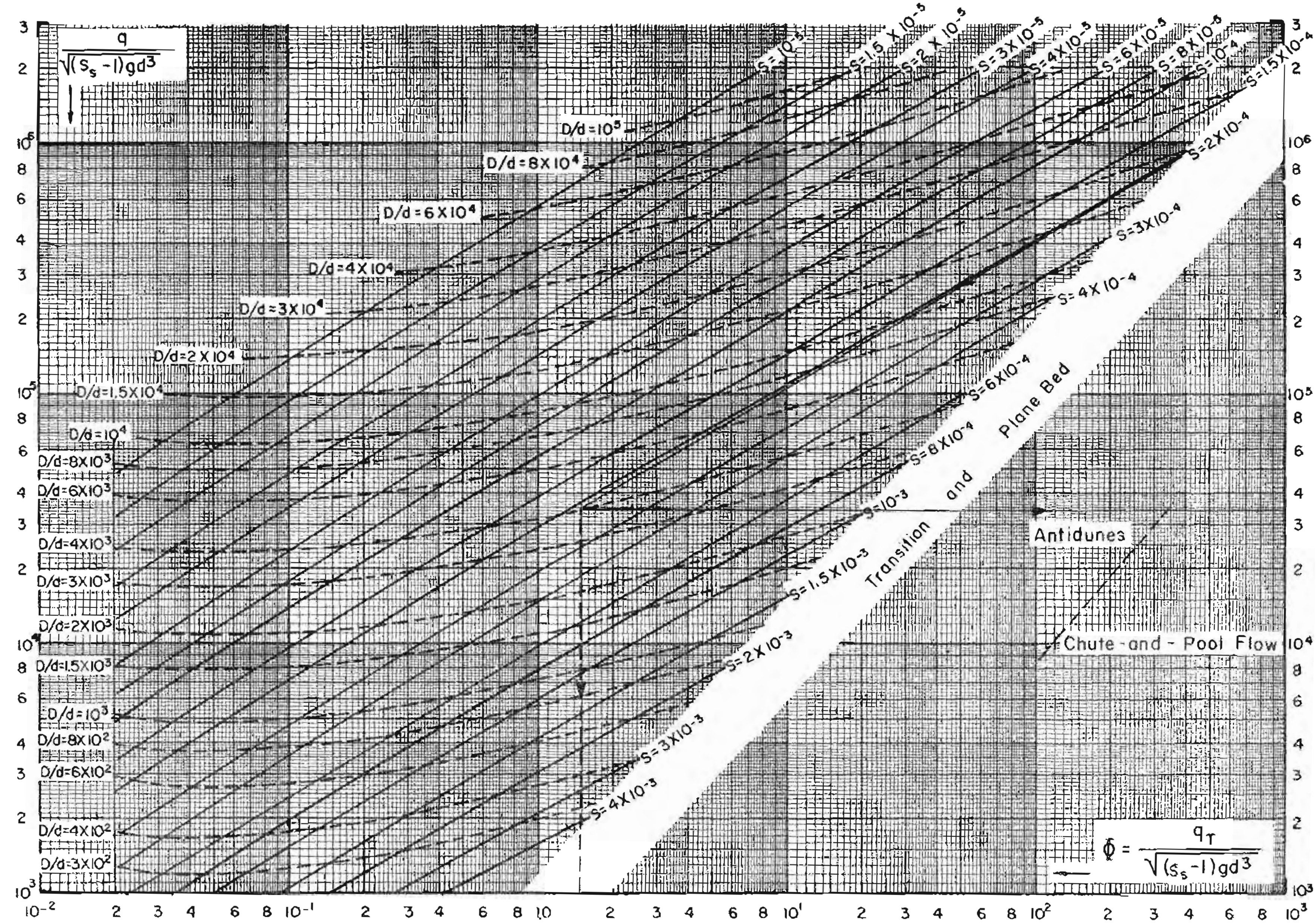

Figure 4–5.—Graphical solution to $\frac{q}{\sqrt{(S_s - 1)gd^3}}$ and ϕ in the Engelund-Hansen procedure. Adapted from Engelund and Hansen (1967) and American Society of Civil Engineers (1975, p. 209).

In summary, the velocity of 3.22 ft/s, discharge of 3.22 $ft^3/(s \cdot ft)$, and bed-material transport of 0.262 $lb/(s \cdot ft)$ of width are determined for a transitional or upper plane-bed regime. The Engelund-Hansen procedure does not provide a means for determining the bed-material discharge at lower flow regimes of plane beds and ripples. These regimes are not significant in terms of the volume of sediment transported.

Colby Procedure for Relating Mean Velocity to Sand Transport

The Colby procedure was developed by correlating mean velocity with sediment concentration in a sand-bed stream. The procedure, partly empirical and partly derived from Einstein's bedload function, is based on measurements in flumes and channels. The relationships are presented in figure 4–6, which gives the uncorrected sand transport as a function of velocity, depth, and the d_{50} particle size of bed material for water depths (D) of 0.1, 1, 10, and 100 ft. Each of the four sets contains curves corresponding to d_{50}'s of 0.10, 0.20, 0.30, 0.40, 0.60, and 0.80 mm.

Before the graphs in figure 4–6 can be used, velocity must be determined by observation or calculation. The bed-material load for flows with a depth other than the four values for which curves are given can be determined by reading the sand transport per foot of width (q_T) for the known velocity for the two depths indicated in figure 4-6 that bracket the desired depth. A log-log plot of D versus q_T enables interpolation of the bed-material load for the desired depth.

This bed-material load corresponds to a water temperature of 60° F and to material with negligible amounts of fine particles in suspension. The two correction factors, K_1 and K_2, in figure 4–7a compensate for the effect of water temperature and concentration of fine suspended sediment on sediment discharge if the d_{50} size of bed sediment is about 0.2 to 0.3 mm. Figure 4–7b represents an estimate of the relative effect of concentration of fine sediment or of water temperature for d_{50} sizes of bed sediment different from those in figure 4–7a. For sizes other than 0.2 and 0.3 mm, multiply the adjustment coefficients from figure 4–7a minus 1.00 by the percentages from figure 4–7b. For example, if an adjustment coefficient (K_1 or K_2) from the main diagram is 1.50 and the d_{50} size of the bed sediment is 0.5 mm, then K_3 from figure 4–7b is 60 percent of 0.5 or 30 percent. The final adjustment coefficient would be 1.30. Colby emphasized that only rough estimates can be derived from figure 4–7.

Using the Graphs to Determine the Discharge of Sands

The discharge of sands in a sand-bed stream can be computed from the graphs as follows:

Example 1, discharge of sands determined from figure 4–6.

Given

Mean velocity	=	5.8 ft/s
Depth	=	8.5 ft
d_{50} size of bed sediments	=	0.26 mm

Figure 4–6 shows that discharges of sands for the given d_{50} size are about 80 and 180 tons/(day·ft) for depths of 1 and 10 ft, respectively. Interpolation using a straightedge for the depth of 8.5 ft on a log-log plot indicates a bed-material discharge of 170 tons per day per foot of width. No corrections are required for temperature, concentration, or sediment size; therefore, the answer is 170 tons.

Example 2, discharge of sands determined from figures 4–6, 4–7a, and 4–7b.

Given

Mean velocity	=	5.8 ft/s
Depth	=	8.5 ft
d_{50} size of bed sediments	=	0.60 mm
Water temperature	=	75°F
Concentration of fine bed sediment	=	20,000 ppm

From figure 4–6, the indicated discharges of sands for the given size of 0.60 mm are about 70 and 110 tons/(day·ft) for depths of 1 and 10 ft, respectively. Interpolation indicates a sand load of 105 tons per day per foot of width for a depth of 8.5 ft. The adjustment coefficient for 75° F (K_1) on figure 4–7a is 0.85 and that for a fine suspended-load concentration of 20,000 ppm (K_2) is 1.55. According to figure 4–7b, the effect of sediment size is only 40 percent as great for a diameter of 0.60 mm as it is for a diameter of 0.20 or 0.30 mm. Therefore, 40 percent of $(1.55-1.00) = 0.22$. The value

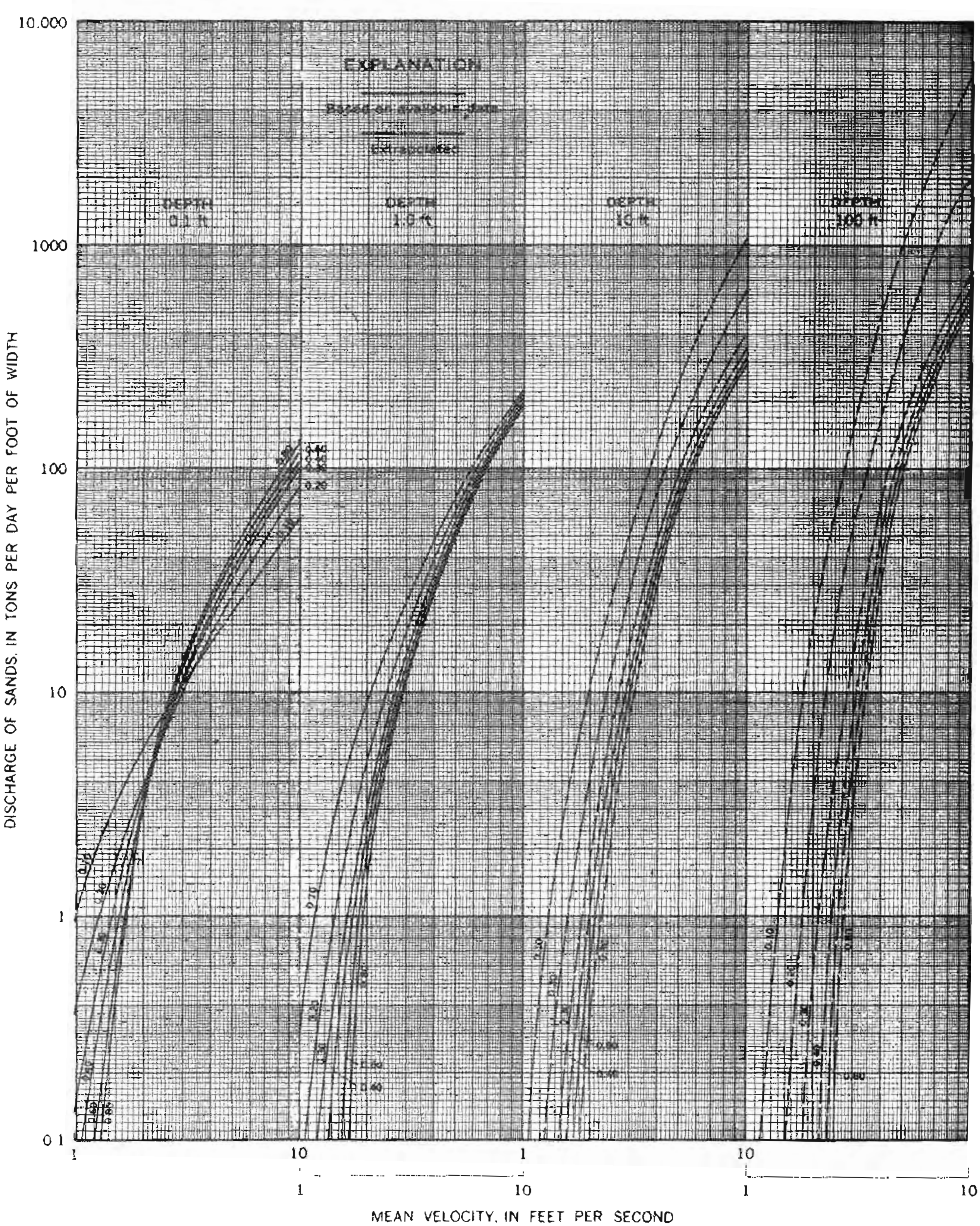

Figure 4-6.—Relationship of discharge of sands to mean velocity for six median sizes of bed sand, four depths of flow, and a water temperature of 60° F. From Colby (1964) and American Society of Civil Engineers (1975, p. 204).

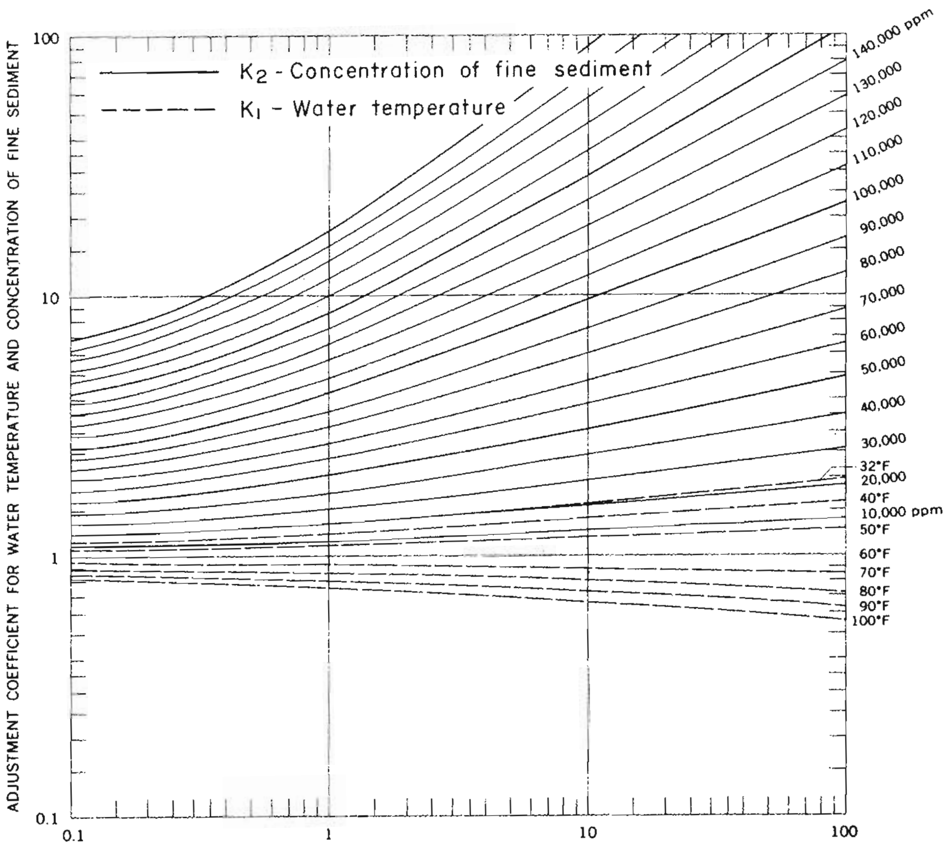

Fig. 4–7a

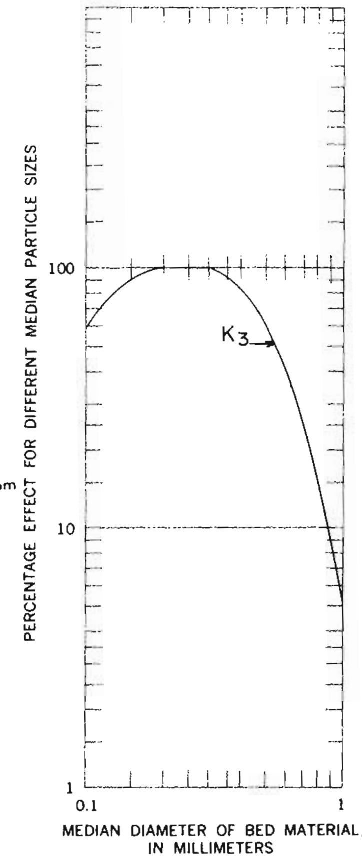

Fig. 4–7b

Figure 4–7.—Approximate correction factors for the effect of water temperature and concentration of fine sediment (4–7a) and sediment size (4–7b) on the relationship of discharge of sands to mean velocity. From Colby (1964) and American Society of Civil Engineers (1975, p. 205).

0.22 is then added to 1.00 to obtain the estimated adjustment coefficient for a diameter of 0.60 mm. The 105 tons/(day·ft) multiplied by 0.85 and by 1.22 gives 109 tons per day per foot of width.

Application and Limitations of Formulas

The lack of certainty in solving specific sediment-transport problems is in part a result of the extremely limited number of situations in which predictive techniques, such as bedload or bed-material transport formulas, have been substantiated by field measurement. Even for techniques that have been substantiated, little information is available about the specific hydraulic characteristics for comparison with conditions for the problem to be solved (Cooper et al. 1972).

Figure 4–8 illustrates a few of the major factors that can be considered in the application and limitations of sediment transport formulas. The availability of bed material ranges from no sand (box A), to an unlimited supply of sand in sizes less than 1 mm (box C), to bed material of gravel and boulders (box E). Flow characteristics range from highly unsteady or rapidly changing to steady and slowly changing.

Of the possible conditions illustrated by this diagram, the condition in box 2C most nearly fits the flow and sediment conditions used in developing transport formulas. Box 1C pertains specifically to the smaller streams with which SCS is concerned, not to rivers in which deep steady flows may transport gravel as they do sand. Through limited reaches and during high flows, shallow streams may also transport gravel and boulders. Frequently there is a transition from scour to deposition over a relatively short reach. Boxes adjacent to 2C (1C, 2B, 2D) can be considered a "gray" area for which correct solutions to sediment transport problems can be obtained by including the appropriate modifiers, such as changes in slope to match variations in discharge.

The effect of rapidly changing flow (top line on the chart) on bedload transport was the subject of a flume study by DeVries (1965). The mean grain size was 2.5 mm. After an equilibrium rate of transport was attained, the tailwater was suddenly lowered while the other factors were kept constant. DeVries computed the lowering of the bed level from scour and the change in rate of sediment transport during the transition to a new state of equilibrium by using several procedures, including the Meyer-Peter and Muller formula. He concluded that establishment and damping of a steady state are slow and that steady-state formulas are unreliable for predicting local, temporary transport for an unsteady state.

A subsequent flume study was made of the effect of introducing a substantial increase (65 percent) in bed-material load into a run where equilibrium flow and transport had been established (Rathbun and Guy 1967). The median size of the sand used was about 0.30 mm. This increase in load increased slope, decreased depth, and increased the transport rate. In another run, the rate of sediment input was reduced to about 50 percent of the equilibrium rate. At first the transport rate was about the same as during equilibrium flow; then, with the degradation of the upper end of the sand bed and the decrease in slope, the transport rate also decreased.

Flow characteristics	No sand A	B	Unlimited supply of sand C	D	Coarse gravel or boulders E
1. Highly unsteady or rapidly changing	1A	1B	1C	1D	1E
2. Steady or slowly changing	2A	2B	2C	2D	2E

Figure 4–8.—Characteristics of bed material.

Aggradation occurs in some channels even though hydraulic computations indicate that sediment should not deposit. It is not always known whether the aggradation occurred in the rising or falling stage of the hydrograph. Some of the unpredicted changes can be explained by variable bed roughness not accounted for in conventional hydraulic computations. Variable bed roughness does not necessarily explain all the inaccuracies in predicting the effects of hydraulic change on sediment transport, however, because some procedures do take into account the changes in bed roughness with various flows. Part of the problem may be due to unsteady flow, since steady-flow procedures fail to account for differences between stage and discharge.

In using computational procedures, it is very important that the supply of bed material just satisfies the capacity for transport under existing hydraulic conditions; that is, there can be neither a deficiency, resulting in scour, nor an excess, resulting in aggradation. A sand bed satisfies the necessary requirements for using bedload or bed-material transport formulas and that of bed-material availability if the bed is sand from bank to bank throughout the reach.

In considering the availability of bed materials, Kellerhals (1966) made a distinction between channels with a sand bed and channels with a gravel bed. According to his studies, channels with a gravel bed cannot be expected to obey the same laws as channels with a sand bed. One distinction is that ripple and dune formation are less significant in channels with a gravel bed.

In terms of particle size, the scarcity of particles in the 2- to 4-mm size fraction, as described by Sundborg (1956), creates a sharp division between predominantly sand-bed streams and predominantly gravel-bed streams. This division has been substantiated by data on sizes of bed material in various parts of the United States.

The segregation of particles in a mixture of sizes, including gravel, and the depth of scour before the formation of armor were the subjects of flume studies by Harrison (1950). The purpose was to determine the most critical condition for segregation and for building an armor during degradation. Harrison used the Einstein bedload function to calculate the limiting grain diameter for equilibrium flow. He determined that a value of ψ (a dimensionless parameter of transport capability) above 27 indicates negligible transport of bed material.

Harrison (1950) found that the representative grain roughness, k_s (assumed to be d_{65} in his procedures), increases during segregation and armor formation. On the basis of data from field and laboratory studies, Kellerhals (1966) computed the k_s values after armor formation to be the d_{90} size.

On the basis of these considerations, the following treatment is suggested for sediment problems in streams as categorized in figure 4–8.

1A, 2A.—For cohesive soil, cemented gravel, and rock, initiation of movement is the important factor in channel scour or bank erosion. Critical tractive force is related to the d_{75} of bank materials. Undisturbed cohesive soil exhibits erosion resistance that may result from one or several characteristics such as structure, permeability, consolidation, cementation, or cohesion. The influence of each of these characteristics has not been identified. Their cumulative effect on erosion resistance, however, can be determined by shear strength tests on undisturbed soil that has been saturated to duplicate moisture conditions during channel flow (Flaxman 1963).

1B, 2B.—A bed only partially covered with sand and exposing different material (cohesive soil, rock, etc.) as the fixed channel boundary indicates a limited sand supply at this specific location. Sediment transport formulas applied to this condition usually yield computed rates that exceed the actual rate. Test the potential for bank erosion by tractive force theory if the bank is composed of noncohesive materials; otherwise, use the procedures for cohesive soils.

1C, 2C.—A sand-covered bed is the condition used in sediment transport formulas if the problem to be solved requires (a) estimating the volume of bed-material transport during a specific interval of time and at a specific level of discharge or (b) comparing the bed-material transport in a reach with that in another reach in which changes in slope, cross section, or discharge may influence the design of a channel. If flow is unsteady, replace the steady-state procedures with the proper unsteady flow relationships, as previously mentioned.

2D.—Techniques for predicting transport rates of sand-gravel mixtures allow estimates of the potential for scour or aggradation. The probable depth of scour can be estimated by determining whether the maximum tractive force for a given flow will exceed the critical for the coarsest 5 to 10 percent of bed material. If the maximum tractive force exceeds the critical for the d_{90} to d_{95}, the depth of scour cannot be predicted unless still coarser material underlies the bed surface material. The amount of scour necessary to develop armor formed of the coarsest fraction can be determined from either the depth of scour or the volume of material removed in reaching this depth.

1D, 1E, 2E.—For gravel and gravel-boulder mixtures, the technique used for determining depth of scour and volume of material produced by scour is similar to that for sand-gravel mixtures (2D). Do not use bedload formulas for this type of material unless confined flow, steepness of slope, and uniformity of cross section provide relatively uniform discharge per foot of width. The highly variable velocity and discharge per foot of width in many alluvial channels is particularly conducive to deposition alternating with scour of coarse bed material.

Conditions favoring bed-material transport at or near a constant and predictable rate do not include delivery in slurries or other forms that change the viscosity and natural sorting processes of flow. Alluvial fills of mountain or foothill canyons are typical of conditions favoring viscous flow. Heavy storm runoff after many years of fill accumulation may produce debris or mud flows whose volume can be predicted only by field measurement.

Comparison of Predictive Methods

Figures 4-9 to 4-11 compare the measured transport rates of bed-material sediment and the predicted rates. The predicted rates were computed by a number of formulas, except that the total bed-material discharge for the Colorado River at Taylor's Ferry (fig. 4-11) was determined from suspended-sediment samples by using the modified Einstein method (U.S. Department of the Interior 1958).

The formula-derived transport rates of bed-material sediment in Mountain Creek (fig. 4-9) follow the general trend of measurements more closely than the comparable rates for the Niobrara and Colorado Rivers (figs. 4-10 and 4-11, respectively). The transport characteristics of Mountain Creek may be more like the flume conditions from which most formulas were derived than like the transport conditions for the two rivers.

In an analysis in *Sedimentation Engineering* (American Society for Civil Engineers 1975), measurements in figures 4-10 and 4-11 were compared with rates computed by several formulas. It was concluded that calculated curves with slopes almost the same as those fitting the data (measurements) are useful even if they do not give the correct values of sediment discharge. Further, although no formula used in figures 4-10 and 4-11 gives lines parallel to those fitting the data, the Colby procedure and the Einstein bedload function consistently gave better results in this regard than the others. It was pointed out that the Colby procedure was derived in part from the Niobrara River data and that the close correspondence between the measured rates and the computed rates could be expected for this reason. Although the analysis included several formulas not described in this handbook, it did not include the Engelund-Hansen procedure, which appears to have merit comparable to that of the Colby and Einstein methods. (The Meyer-Peter or Meyer-Peter and Muller bedload formulas may be applicable for gravel and gravel-boulder mixtures with the limitations for 1D, 1E, and 2E). It appears that appropriate formulas should be used only to relate transport capacity between one reach and another and do not yield dependable quantitative results.

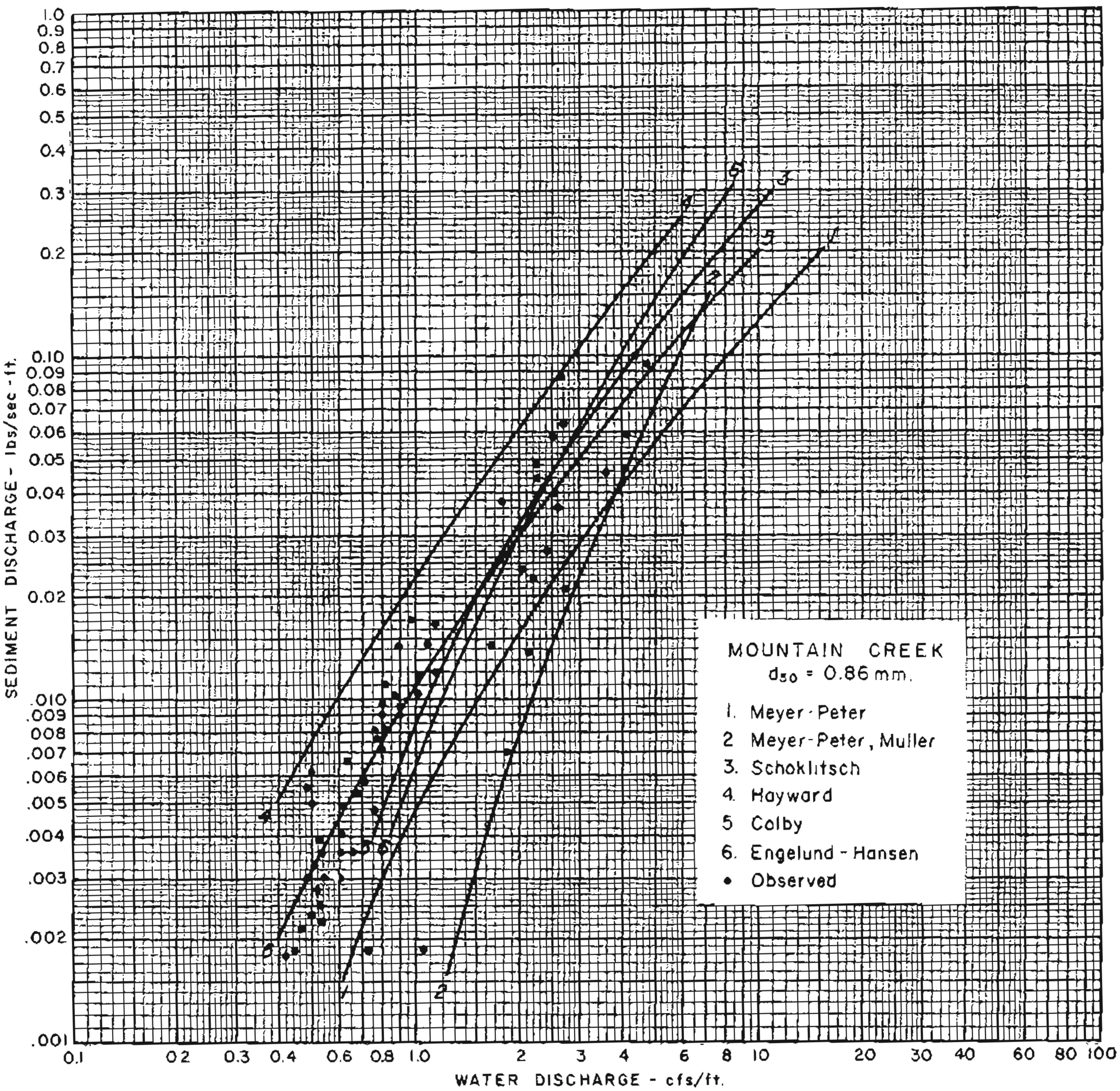

Figure 4–9.—Sediment rating curves for Mountain Creek near Greenville, S.C., according to several formulas compared with measurements. Adapted from Vanoni, Brooks, and Kennedy (1961, p. 7–8).

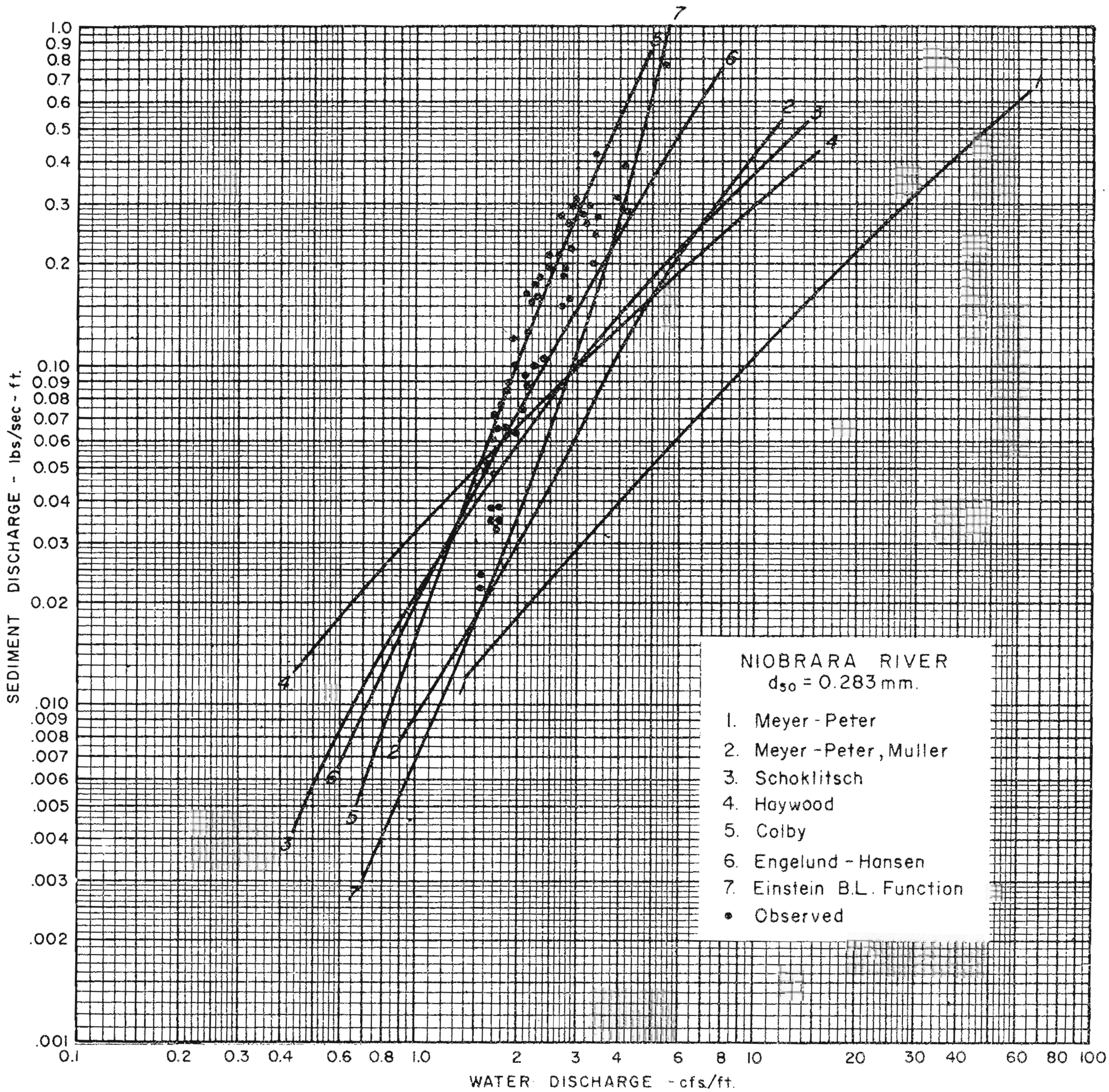

Figure 4-10.—Sediment rating curves for Niobrara River near Cody, Nebr., according to several formulas compared with measurements. Adapted from Vanoni, Brooks, and Kennedy (1961); American Society of Civil Engineers (1975, p. 221).

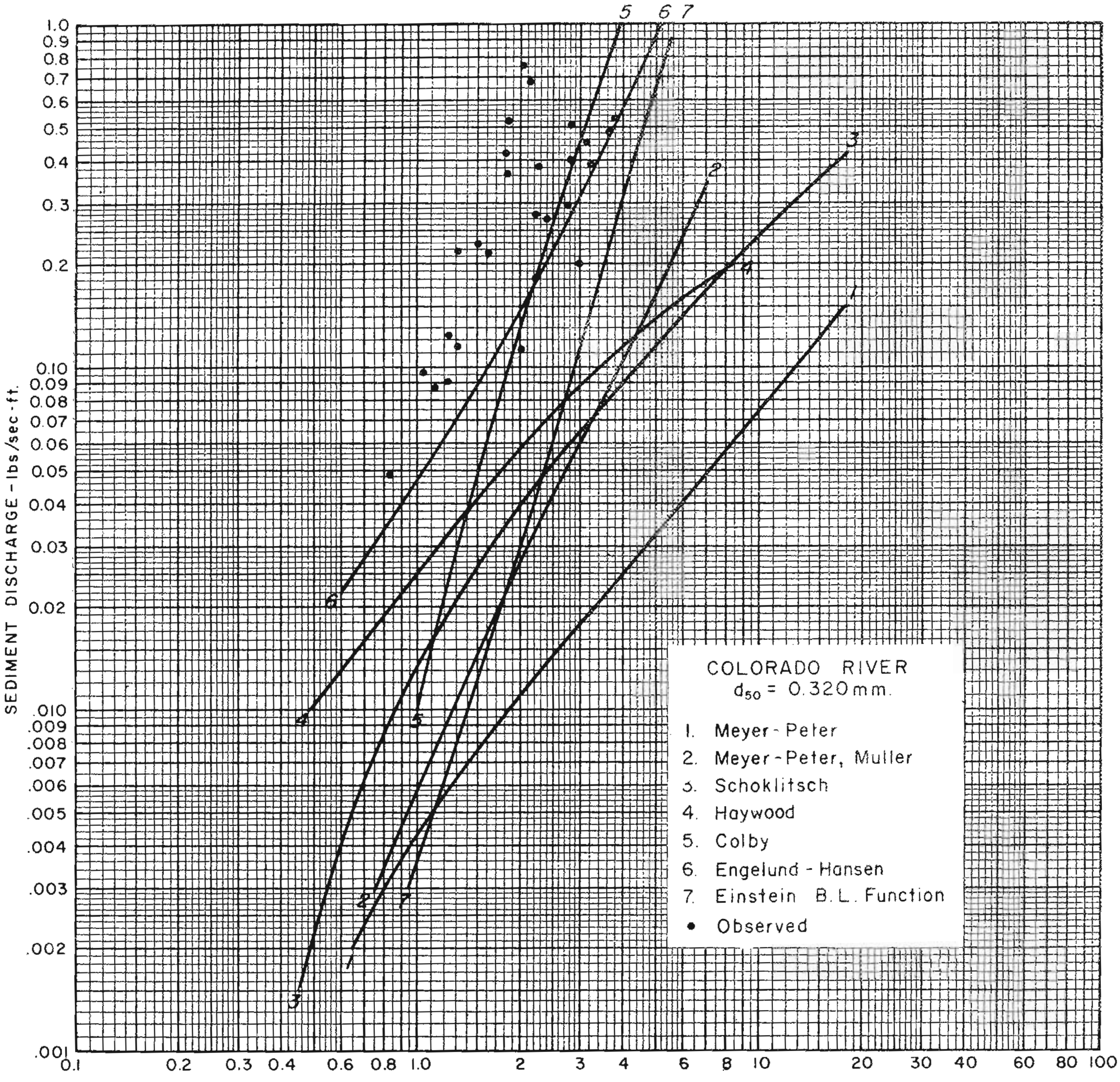

Figure 4–11.—Sediment rating curves for Colorado River at Taylor's Ferry, Ariz., according to several formulas compared with measurements. Adapted from Vanoni, Brooks, and Kennedy (1961); American Society of Civil Engineers (1975, p. 221).

Example of a Channel Problem

The following example illustrates the similarities and differences in results obtained by applying two procedures to determine sediment transport capacity: the Schoklitsch formula and the Colby procedure.

An existing channel 20 ft wide having a bed slope of 0.002 ft/ft has inadequate capacity for controlling flooding of adjacent lands. It has been proposed that the width of this channel be increased to 30 ft to provide the necessary capacity. Field investigations show that an unlimited supply of sand is available for transport in the bed of the channel and that this sand has a d_{50} size of 0.30 mm. Water temperature is 60° F, and the concentration of fine sediment does not exceed 5,000 ppm.

For purposes of simplification, it is assumed that the banks have no effect on depth-discharge relationships. But the roughness of the banks and differences in roughness of the banks in both unimproved and improved reaches can in fact affect depth and velocity for a given discharge and thereby affect the rate of bed-material transport. The hydraulics of the flow, which includes distribution of shear on the banks as well as on the bed, must be determined by an established procedure before computing the bed-material transport.

The hydrograph used in this example is divided into segments to determine the discharge per foot of stream width as required for the computational procedures. The mean discharge and duration for each of the hydrograph segments are shown in table 4–1.

Table 4–1.—Discharge data for example channel problem, high flow

Hydrograph segment	Discharge per foot of width	
	20-ft channel	30-ft channel
	ft³/s	*ft³/s*
Rising stage:		
a. Mean flow for 2 hours, 90 ft³/s	4.5	3.0
b. Mean flow for 2 hours, 280 ft³/s	14.0	9.333
Falling stage:		
c. Mean flow for 3 hours, 240 ft³/s	12.0	8.0
d. Mean flow for 3 hours, 180 ft³/s	9.0	6.0
e. Mean flow for 3 hours, 40 ft³/s	2.0	1.333

The Schoklitsch formula requires data only for the amount of discharge per foot of width. The Colby procedure requires velocity and depth of flow. To determine velocity and depth for a given discharge (unless they are available from streamgage records), it is necessary either to assume an "n" roughness coefficient for use in the Manning equation or to obtain such values empirically. For solution of the example problem by the Colby procedure, two approaches are used. In one, a constant assumed "n" of 0.020 is used. In the other, the most recent and perhaps the most reliable procedure (Alam and Kennedy 1969) for predicting friction factors (and thereby depth, velocity, and discharge relationships) is used. See the appendix to this chapter for details of this procedure.

The data in table 4–2 indicate that in the stated problem the Schoklitsch formula predicts considerably less sediment transport than either of the Colby approaches. This difference may be due to the fact that the Schoklitsch formula predicts bedload and the Colby procedure accounts for suspended bed material as well as bedload. The difference between the two Colby predictions can be attributed to the different approaches for estimating the depth of flow. The first assumes n = 0.020 and a normal depth based on bed slope equal to friction slope; the second assumes a normal depth based mostly on grain roughness for friction slope.

The Alam and Kennedy friction factors are never in the lower flow regime for this set of calculations; therefore, bedform changes had little effect on the results. All three results indicate a slight, but negligible, reduction (less than 5 percent) in sediment transport capacity for the wider channel.

The next step in the analysis is to determine whether lower flows would give different results. For this computation, 20 percent of the discharges indicated in table 4–1 are used in table 4–3.

Table 4–4 shows the amount of sediment transported as computed by the two procedures. Table 4–4 again indicates considerable difference between the Schoklitsch and Colby predictions, but less than that shown in table 4–2. This smaller difference can be attributed to the smaller loads in suspension for the lower flows. All three predictions, however, indicate greatly reduced sediment transport capacity for the wider (30-ft) channel (9, 17, and 32 percent, respectively). The most significant reduction, almost one-third, is predicted by the Colby procedure using the Alam and Kennedy friction factors. It is believed that the Colby procedure

Table 4-2.—Sediment transport computed for various flows

Discharge segment	Schoklitsch formula 20-ft width	Schoklitsch formula 30-ft width	Colby procedure: Using n = 0.020 20-ft width	Colby procedure: Using n = 0.020 30-ft width	Colby procedure: Using Alam and Kennedy friction factors 20-ft width	Colby procedure: Using Alam and Kennedy friction factors 30-ft width
	lb	*lb*	*lb*	*lb*	*lb*	*lb*
a	44,135	42,840	97,285	86,720	109,270	103,225
b	142,760	141,470	347,085	344,210	412,425	543,140
c	182,995	181,060	442,745	426,435	590,170	564,565
d	136,280	134,340	328,735	310,100	516,280	431,920
e	27,270	25,330	50,710	42,765	46,180	31,190
Total	533,440	525,040	1,226,560	1,210,230	1,674,325	1,674,040
Ratio $\left(\frac{\text{20-ft width}}{\text{30-ft width}}\right)$	$\frac{525{,}040}{533{,}440}$ = 98.43 percent		$\frac{1{,}210{,}230}{1{,}266{,}560}$ = 95.55 percent		$\frac{1{,}674{,}040}{1{,}674{,}325}$ = 99.98 percent	

Table 4-3.—Discharge data for example channel problem, lower flow

Hydrograph segment	Discharge per foot of width: 20-ft channel	Discharge per foot of width: 30-ft channel
	ft³/s	*ft³/s*
Rising stage:		
a. Mean flow for 2 hours, 18 ft³/s	0.9	0.6
b. Mean flow for 2 hours, 56 ft³/s	2.8	1.87
Falling stage:		
c. Mean flow for 3 hours, 48 ft³/s	2.4	1.6
d. Mean flow for 3 hours, 36 ft³/s	1.8	1.2
e. Mean flow for 3 hours, 8 ft³/s	0.4	0.267

using the Alam and Kennedy factors most closely reflects the influence of variable bed forms that are more pronounced during low to moderate flows.

This example clearly shows that estimates of the absolute rates of sediment transport vary according to the procedure. But the study also shows that the relative rates can be insensitive to choice of procedure if variation in bed forms is not a factor, as for channel performance at peak discharge. In many stability problems, however, the performance of the channel during one or more low to moderate flows must be considered. Formulas and procedures that determine the effect of variable bed forms on depth, velocity, and discharge relationships and, thereby, on bed-material discharge afford greater flexibility for all purposes.

Table 4–4.—Sediment transport computed for lower flows

Discharge segment	Schoklitsch formula		Colby procedure			
			Using n = 0.020		Using Alam and Kennedy friction factors	
	20-ft width	30-ft width	20-ft width	30-ft width	20-ft width	30-ft width
	lb	*lb*	*lb*	*lb*	*lb*	*lb*
a	6,760	5,470	9,970	7,195	450	700
b	26,485	25,195	53,280	46,705	61,225	41,645
c	33,500	31,560	67,580	54,615	66,255	46,245
d	24,155	22,220	43,710	36,000	39,245	24,500
e	2,355	415	3,315	2,525	940	415
Total	93,255	84,860	177,855	147,040	168,115	113,505
Ratio $\left(\frac{\text{30-ft width}}{\text{20-ft width}}\right)$	$\frac{84{,}860}{93{,}255}$ = 91.00 percent		$\frac{147{,}040}{177{,}855}$ = 82.67 percent		$\frac{113{,}505}{168{,}115}$ = 67.52 percent	

Summary of Procedures for Evaluating Bed-Material Transport Problems

Problems of bed-material transport require consideration of three elements: (1) existing conditions, (2) availability of bed material, and (3) natural or artificial changes in stream or watershed conditions. The existing conditions can be best determined by field investigation and analysis. Surveys of old and new cross sections, use of techniques for identifying depth of scour or aggradation, and comparison of aerial photographs all facilitate definition of the problems.

Although the correct identification and analysis of existing bed-material transport conditions are important, most problems require projections of what will or can occur rather than what is now occurring. The availability of bed material and the impact of change are the key elements of such projections.

Equilibrium can be achieved only if bed material is being introduced into the reach at a rate comparable to that at which bed material moves out of the reach. Problems arise when the amount introduced is greater or less than the transport capacity of the flow. In other words, equilibrium transport seldom causes problems but a change from equilibrium to nonequilibrium transport often does.

The supply of bed material can exceed transport capacity during unusually high discharges. This excess can be caused by development of new and substantial sources of bed material within or adjacent to the problem reach or by channel changes that may increase transport capacity in the upstream reach but not in the downstream reach. Determing the availability of bed material is largely a field problem. To be readily available to channel flow, sediment must be in the stream system. The coarse particles in an upland soil tend to lag behind during erosion. Gullies that feed directly into the stream system and that expose soils with a large proportion of particles of bed-material size can be major contributors but do not in themselves constitute an immediate and unlimited stream channel supply.

Streambanks that have, at least in part, soil textures comparable to those in the bed, can be a ready source of supply, depending on the ease with which the flow can erode the material. A frequently used emergency flood-protection measure is to bulldoze streambed materials to each side to form banks or levees. These banks are a ready source of supply. Their erosion and the consequent deterioration of channel alignment result in overloading the flow and downstream aggradation.

Scour of bed material can result from an undersupply of sediment in an alluvial reach. Upstream changes in watershed or stream conditions that can reduce the supply of incoming bed material include the removal of supply by major flood scour and the construction of reservoirs, debris basins, or other structures.

In addition to cutting off the supply of bed material to the reach downstream, a reservoir can materially influence the stability of the channel bed and banks by modifying the flow. For example, a detention structure that controls a high flood peak can thereby extend the duration of released flows by days. The resulting bed and bank scour may be extensive because of the energetic discharge of clear water.

Table 4–5 is a checklist of procedures to consider in solving problems of bed-material transport. The last column in this table indicates that a field evaluation is important to the solution of any such problem. Because of the variety of factors that can influence their solution, most problems are not routine and solving them requires the assistance of well-trained and experienced personnel. The first step should always be a field evaluation of existing or potential problems related to sediment transport. With experience, well-trained personnel frequently can find answers to questions of stability, degradation, or aggradation by relating the availability of bed material to proposed changes in the hydraulics of the flow without resorting to formulas. If formulas must be used, it should be recognized that the results are qualitative and not quantitative. Observations of similar streams having comparable drainage areas, geology, soils, topography, and runoff often provide guidance on the probable stability.

Table 4–5.—Checklist of procedures for solving bed-material transport problems

Item	Analysis procedure: Tractive stress[1]	Comparative hydraulics[2]	Bed material formulas	Field evaluation
Problem characteristics:				
Erodibility of bed	x			x
Erodibility of bed and banks	x			x
Erodibility of banks	x			x
Channel aggradation		x	x	
Volume of bed material			x	x
Effects of channel change		x	x	x
Channel boundary characteristics:				
Cohesive soils	x			x
Cohesive soils or rock with intermittent deposits of sand or gravel	x			x
Sand ≤1.0 mm	x	x	x	x
Sand ≤1.0 mm with <10% gravel	x	x	x	x
Gravel, gravel mixed with sand	x		[3]	x
Gravel and boulders	x		[3]	x
Hydraulic characteristics:				
In problem reach:				
Steady state or slowly changing	x	x	x	x
Rapidly changing	x	x		x
Cross section—slope upstream vs problem reach:				
About the same	x	x	x	x
Steeper slope	x	x	x	x
Wider channel	x	x	x	x
Narrower channel	x	x	x	x

[1]For cohesive soil boundaries, analysis may include tractive power (tractive stress times mean velocity).
[2]Comparison of relationships between depth, velocity, and unit discharge in two or more reaches.
[3]Special situations, see page 4–19.

Suspended-sediment load includes both the bed-material load in suspension and the wash load, as shown in figure 4–2. If erosion of fine-texured soils is the chief source of sediment, the wash load, not the bed-material load, usually constitutes the bulk of the sediment discharge. No method exists for predicting rates of wash-load transport unless there is a substantial amount of data on concentrations of suspended sediment during measured discharges.

Suspension Mechanism

Bagnold (1966) explains the suspension mechanism as follows:

> Isotropic turbulence cannot by definition be capable of exerting any upward directed stress that could support a suspended load against gravity. For any suspended solid must experience over a period of time a downward flux of eddy momentum equal on the average to the upward flux. A swarm of solids would be dispersed equally in all directions by diffusion along uniform concentration gradients, but the center of gravity of the swarm would continue to fall toward a distant gravity boundary.
>
> The center of gravity of a swarm of solids suspended by shear turbulence, on the other hand, does not fall toward the gravity shear boundary. The excess weight of the solids remains in vertical equilibrium. It follows therefore that the anisotropy of shear turbulence must involve as a second-order effect a small internal dynamic stress directed perpendicularly away from the shear boundary. In other words, the flux of turbulent fluid momentum away from the boundary must exceed that toward it. . . . The turbulence appears to be initiated and controlled by a process akin to the generation of surface waves by a strong wind. An upwelling on the part of a minor mass of less turbulent boundary fluid intrudes into an upper, faster moving layer, where its crest is progressively torn off, like spray, and mingles with the upper layer. Corresponding motion in the reverse sense are [sic] absent or inappreciable.
>
> Since there cannot be a net normal transport of fluid, the return flow must be effected by a general sinking toward the boundary on the part of a major mass of surrounding fluid.

The settling rate for sediment particles of uniform density increases with size, but not proportionally. The settling rate for particles smaller than about 0.062 mm varies approximately as the square of the particle diameter, whereas particles of coarse sand settle at a rate that varies approximately as the square root of the diameter. The settling rate for particles of intermediate size varies at an intermediate rate. The dividing line between sediments classed as silts and those classed as sands is the 0.062-mm size. Clay and silt particles usually are distributed fairly uniformly in a stream, but sand particles usually are more concentrated near the bottom. The degree of variation is a function of the coarseness of the particle (fig. 4–12).

The lateral distribution of suspended sediment across a stream is fairly uniform in both deep and shallow flows except below the junction of a tributary carrying material at a concentration substantially different from that of the main stream. The flow from the tributary tends to remain on the entrance side of the channel for some distance downstream.

Sampling and Laboratory Procedures

The U.S. Geological Survey collects most of the suspended-sediment samples in this country. Samples are collected by lowering and raising an integrating sampler vertically in the flow at a uniform rate. Travel time to and from the streambed is regulated so that the container is not quite full of the water-sediment mixture when it returns to the surface. This regulation provides uniform sampling for the sampled depth of flow. Flows are sampled to within about 4 in. of the bed.

Point-integrating samplers have a tripping mechanism that enables sampling at any point in the flow. Data on concentration and composition of the bed material are used in computing the total bed-material load. Point-integrating samplers are sometimes used in streams too deep for equipment that can collect integrated samples only. Sixteen feet is about the maximum depth for obtaining integrated samples.

Laboratory procedures used in handling the samples include weighing the container holding the water-sediment mixture and then decanting the clear liquid, evaporating the remaining moisture, and weighing the dry sediment. The ratio of the dry weight of the sediment times 10^6 to the weight of the water-sediment mixture is the sediment concentration in parts per million. The suspended-sediment concentration can be experssed in milligrams per liter by using the following formula (American Society of Civil Engineers 1975, p. 403).

$$\text{Concentration in milligrams per liter} = A\left(\frac{\text{weight of sediment} \times 10^6}{\text{weight of water-sediment mixture}}\right) \quad 4\text{–}6$$

Factor A is given in table 4–6.

Suspended-sediment load stations can be classified according to how often they collect and report data. Stations reporting daily can collect several samples during a high or variable discharge. Periodic stations collect samples about every 2 weeks or less frequently. Daily stations report mean discharge, sediment concentration in tons, and a summation of the latter for the month and year. Periodic stations usually report data for only the day of sampling. Size distribution is frequently obtained for representative samples.

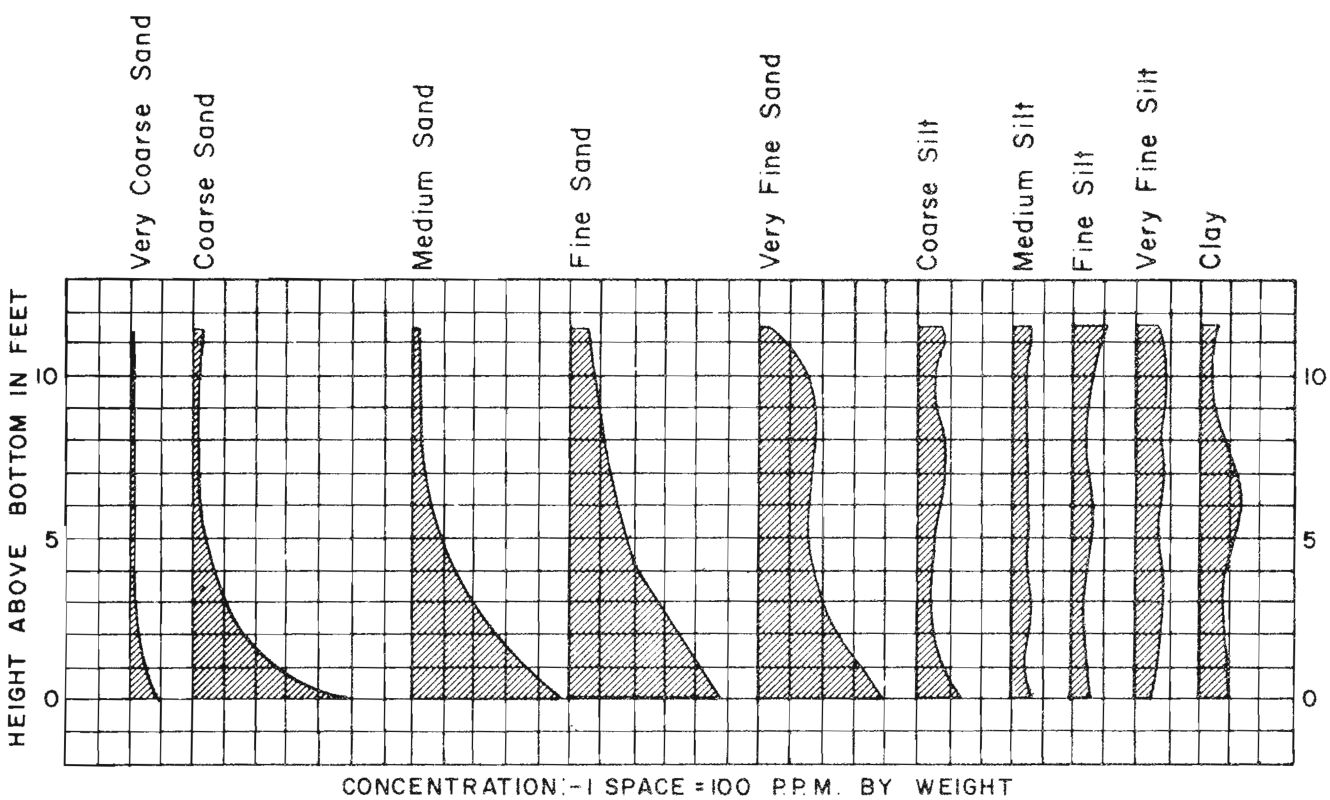

Figure 4–12.—Vertical distribution of sediment in Missouri River at Kansas City, Mo. From Federal Inter-Agency River Basin Committee (1963, p. 28).

Table 4-6.—Factor A for computing sediment in milligrams per liter by equation 4-6

$\frac{\text{Wt of sediment}}{\text{Wt of sediment-water mixture}} \times 10^6$	A	$\frac{\text{Wt of sediment}}{\text{Wt of sediment-water mixture}} \times 10^6$	A
0 - 15,900	1.00	322,000 - 341,000	1.26
16,000 - 46,900	1.02	342,000 - 361,000	1.28
47,000 - 76,900	1.04	362,000 - 380,000	1.30
77,000 - 105,000	1.06	381,000 - 398,000	1.32
106,000 - 132,000	1.08	399,000 - 416,000	1.34
133,000 - 159,000	1.10	417,000 - 434,000	1.36
160,000 - 184,000	1.12	435,000 - 451,000	1.38
185,000 - 209,000	1.14	452,000 - 467,000	1.40
210,000 - 233,000	1.16	468,000 - 483,000	1.42
234,000 - 256,000	1.18	484,000 - 498,000	1.44
257,000 - 279,000	1.20	499,000 - 513,000	1.46
280,000 - 300,000	1.22	514,000 - 528,000	1.48
301,000 - 321,000	1.24	529,000 - 542,000	1.50

If daily or more frequent data on the concentration of suspended sediment are available, tons per day can be computed by plotting the concentration directly on a chart showing gage height against time. Draw a smooth curve through the plotted points and read the daily mean concentration from the graph. If data on rapidly changing concentration and water discharge are available, divide the graphs into smaller increments of time (American Society of Civil Engineers 1975, p. 345).

Sediment-Rating Curve and Flow-Duration Curve Method of Computing Suspended-Sediment Load

Periodic data on suspended sediment or short-term daily data are sometimes extended for use as average annual yeilds by constructing sediment-transport rate and flow-duration curves. A sediment-transport rate curve constructed by plotting discharge and sediment-load data in tons is shown in figure 4–13. It is not essential to plot all the data available, but plot enough over a wide range of discharges to be able to draw a curve that will cover and perhaps extend the range of data.

To construct a flow-duration curve, divide data on mean discharges into a series of classes over a range that has been recorded at this station. Then, count the number of days within each class. Determine the percentage of time in each class and plot the midpoint on log-probability paper against the accumulated percentage at that point. Figure 4–14 is an example of a flow-duration curve. Table 4–7 illustrates how to use the sediment-transport rate curve and the flow-duration curve to determine the annual sediment yield for the period on which the flow-duration curve is based. Construction of this particular curve is based on the total number of days of record. Each segment of the curve represents the proportion of a composite day in which a particular flow occurs during the period of record. For example, in figure 4–14 discharge is 100 ft^3/s or greater for 10 percent of a composite day. Methods of preparing flow-duration curves are described in detail by Searcy (1959).

The figures in column 1, table 4–7, refer to segments of the flow-duration curve; for example, the entries in horizontal line 1 are for the segment between 0.01 percent and 0.05 percent of the composite day.

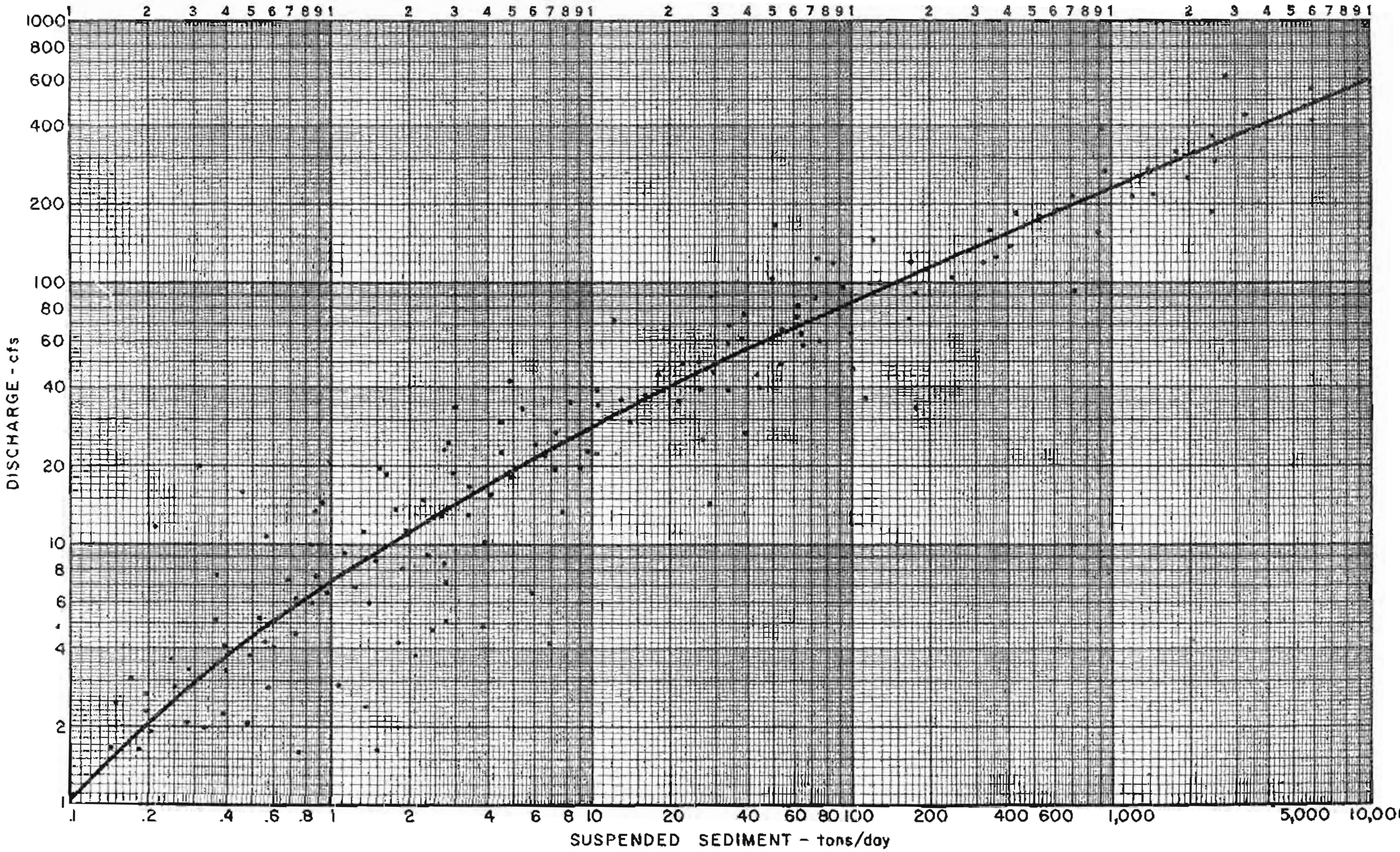

Figure 4-13.—Sediment rating curve, Cottonwood Creek, any State.

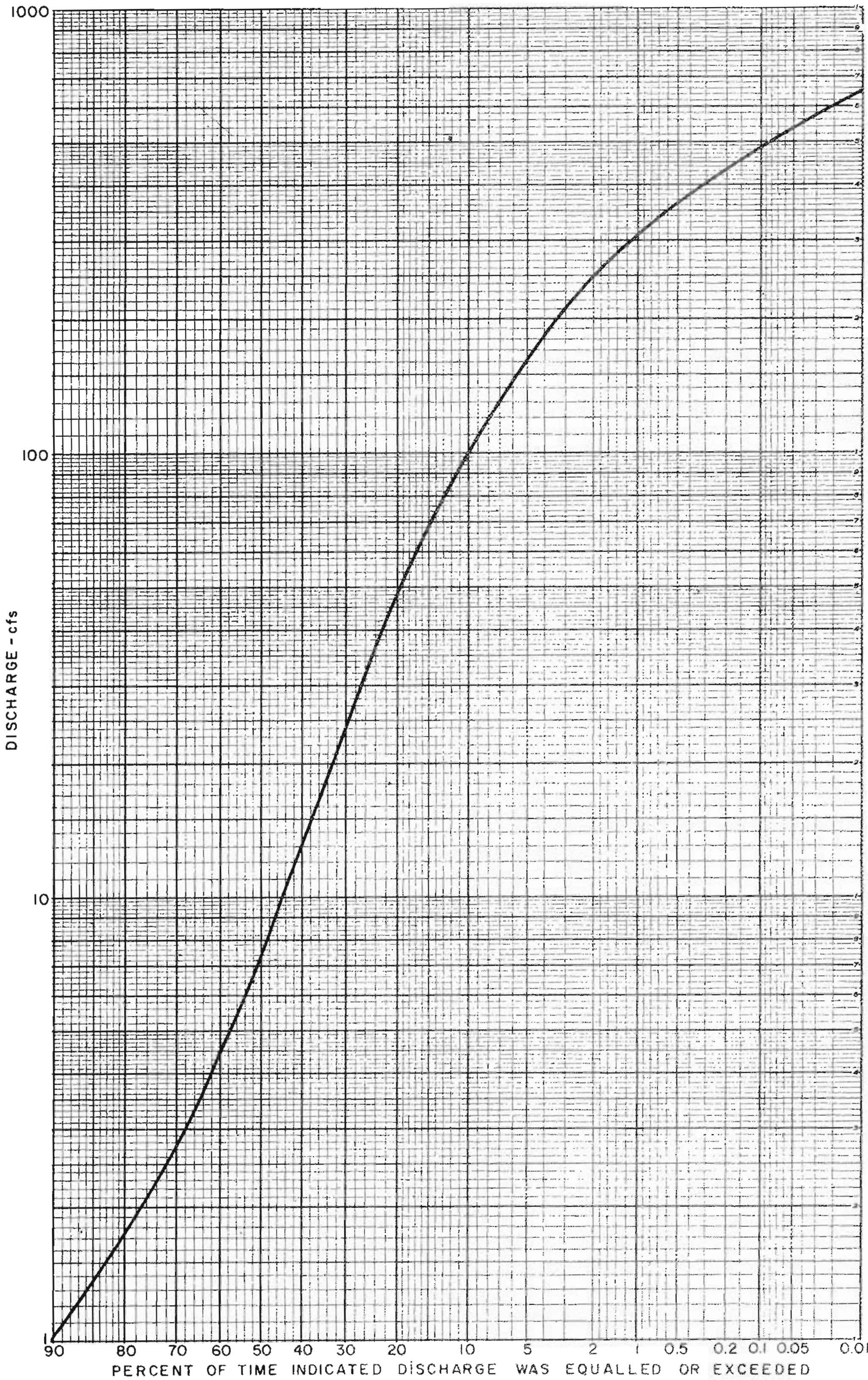

Figure 4-14.–Flow-duration curve, Cottonwood Creek, any State.

Table 4-7.—Computation of average annual suspended-sediment load, Cottonwood Creek, Any State

1 Percentage limits	2 Percentage interval	3 Percentage (mid ordinate)	4 Discharge Q_w	5 Sediment load, Q_s	6 Discharge (Q_w) per day Col. 2 x Col. 4	7 Sediment load (Q_s) per day Col. 2 x Col. 5
			ft³/s	*tons*	*ft³/s*	*tons*
0.01 - 0.05	0.04	0.030	590	9,000	0.24	3.6
0.05 - 0.1	0.05	0.075	505	6,400	0.25	3.2
0.1 - 0.5	0.4	0.30	400	3,500	1.6	14.0
0.5 - 1.5	1.0	1.0	310	1,900	3.1	19.0
1.5 - 5	3.5	3.25	200	700	7.0	24.5
5 - 15	10	10	100	145	10.0	14.5
15 - 25	10	20	47	28	4.7	2.8
25 - 35	10	30	25	8	2.5	0.8
35 - 45	10	40	13	3	1.3	0.3
45 - 55	10	50	7	1	0.7	0.1
55 - 65	10	60	4	0.5	0.4	0.05
65 - 75	10	70	3	—	0.3	—
75 - 85	10	80	2	—	0.2	—
85 - 95	10	90	1	—	0.1	—
				Total	32.39	82.8

Annual sediment load = 82.8 x 365.25 = 30,240 tons

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Appendix: Derivation of Friction Factors for Flow in Sand-Bed Streams by the Alam-Kennedy Procedure

The following procedure was used to determine depth-discharge relationships for the problem described on pages 4–24 to 4–26. The procedure is empirical and is designed to reflect the influence of variable bed roughness on flow and thus on sediment transport. The hydraulic conditions were described briefly on pages 4–8 and 4–9. By comparing observed depth-discharge relationships with predicted relationships, Alam and Kennedy (1969) demonstrated that the procedure applies to the full spectrum of bed forms. They considered depth equivalent to hydraulic radius, an assumption that must be adjusted for channels having substantial differences between the two factors. In addition, the effect of bank roughness should be evaluated.

As illustration of the Alam-Kennedy procedure, the computations for deriving depth-discharge curves are given in the following example. These curves were used to determine sediment transport (tables 4–2 and 4–4) for the Colby method with the Alam-Kennedy technique.

As in the problem presented on pages 4–24 to 4–26, the bank influence is assumed to be negligible so that the hydraulic radius (R) is assumed to be equal to the hydraulic radius with respect to the bed (R_b).

Given:

$$\text{channel slope} = 0.002 \text{ ft/ft}$$
$$d_{50} \text{ size of bed material} = 0.3 \text{ mm} = 0.000984 \text{ ft}$$

For a velocity of 3.5 ft/s, calculate the Froude number where

$$F_D = \frac{U}{\sqrt{gd_{50}}} = \frac{3.5}{0.178} = 19.66$$

Assume $R_b = 1.30$ ft

$$\frac{R_b}{d_{50}} = \frac{1.30}{0.000984} = 1321$$

$$\nu = 1.22 \times 10^{-5} \text{ ft}^2/\text{s (for } 60^\circ \text{ F)}$$

$$R_N = \frac{UR_b}{\nu} = \frac{3.5(1.3)}{1.22 \times 10^{-5}} = 3.73 \times 10^5$$

From figure 4–15, using the values of $U/\sqrt{gd_{50}}$ and R_b/d_{50}, obtain f_b'' (Darcy-Weisbach bed-form friction factor):

$$f_b'' = 0.025$$

From figure 4–16 (Lovera and Kennedy 1969) obtain f'_b (flat-bed friction factor), using the values of R_N and R_b/d_{50}:

$$f'_b = 0.017$$

The total friction factor, f_b, $= f'_b + f''_b = 0.017 + 0.025 = 0.042$.

Calculate the hydraulic radius:

$$R_b = f_b \frac{U^2}{8gS} = \frac{0.042(3.5)^2}{8g(0.002)} = 1.00$$

Because the calculated and assumed values differ by an excessive amount, repeat the preceding steps, using the new value of R_b:

$$\frac{R_b}{d_{50}} = \frac{1.00}{0.000984} = 1{,}016$$

$$R_N = \frac{UR_b}{\nu} = \frac{3.5(1.00)}{1.22 \times 10} = 2.87 \times 10^5$$

From figure 4–15, $f''_b = 0.0215$. From figure 4–16, $f'_b = 0.020$. Then $f_b = f'_b + f''_b = 0.020 + 0.0215 = 0.0415$

$$R_b = f_b \frac{U^2}{8gS} = \frac{0.0415(3.5)^2}{8g(0.002)} = 0.987$$

Because the difference between the calculated and last assumed value of R_b is less than 2 percent, additional computation is unjustified.

$$R_b = 1.00$$

$$U = 3.5$$

$$q = R_bU = 3.5 \text{ ft}^3/(\text{s} \cdot \text{ft})$$

These steps were repeated for velocities between 1.0 and 7.0 ft/s to provide data for the R_b-velocity curve in figure 4–17. The R_b-discharge curve in figure 4–17 was then plotted. Both curves were then used in the derivations of the Colby procedure to yield sediment transport data shown in tables 4–2 and 4–4.

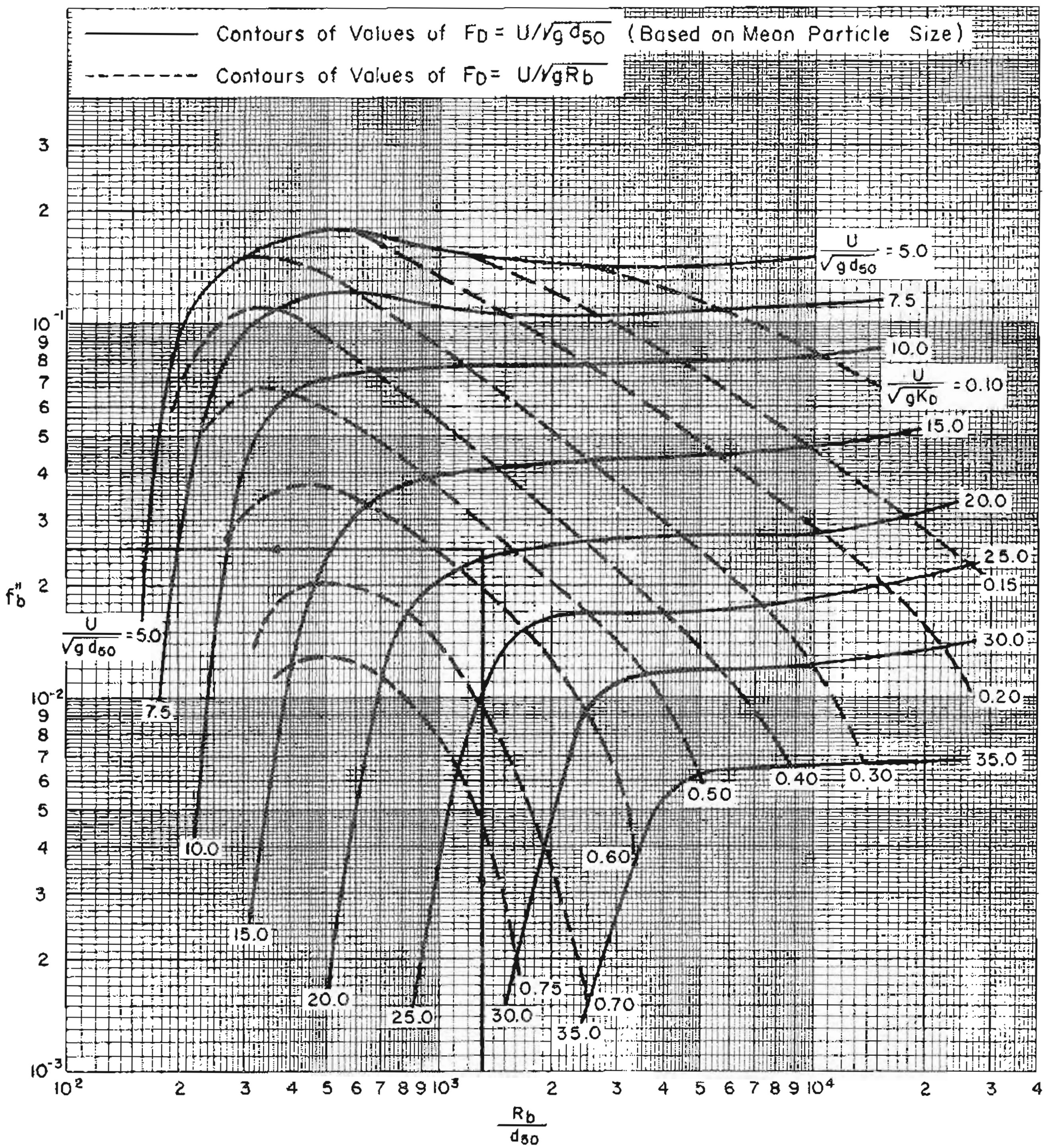

Figure 4-15.—Form-drag friction factor in sand-bed channels, f_b'', as a function of R_b/d_{50} and $F_D = U/\sqrt{g d_{50}}$. From Alam and Kennedy (1969), American Society of Civil Engineers (1975, p. 142).

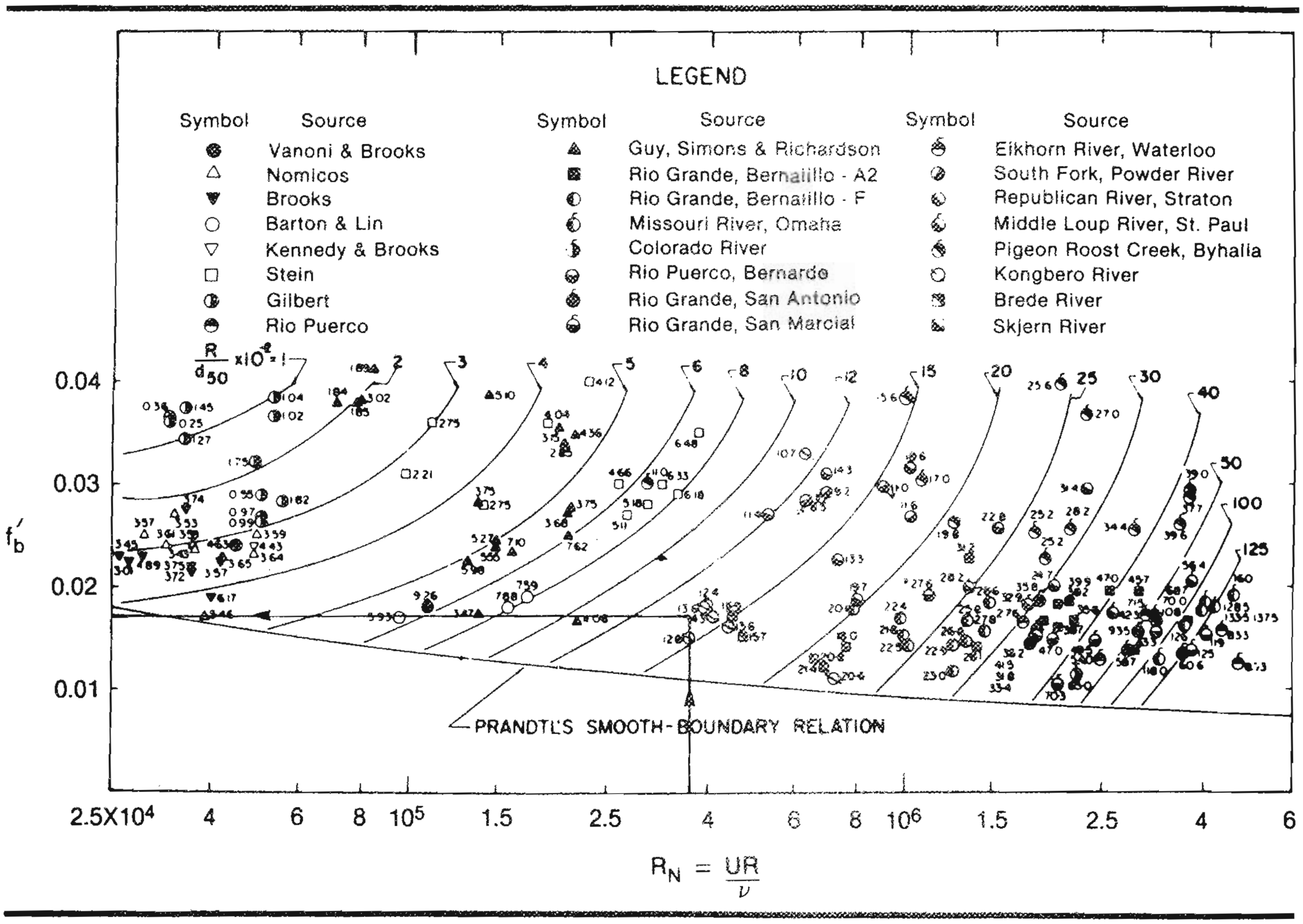

Figure 4–16.—Friction-factor predictor for flat-bed flows in alluvial channels. The number by each point is $R/d_{50} \times 10^{-2}$. From Lovera and Kennedy (1969), American Society of Civil Engineers (1975, p. 140).

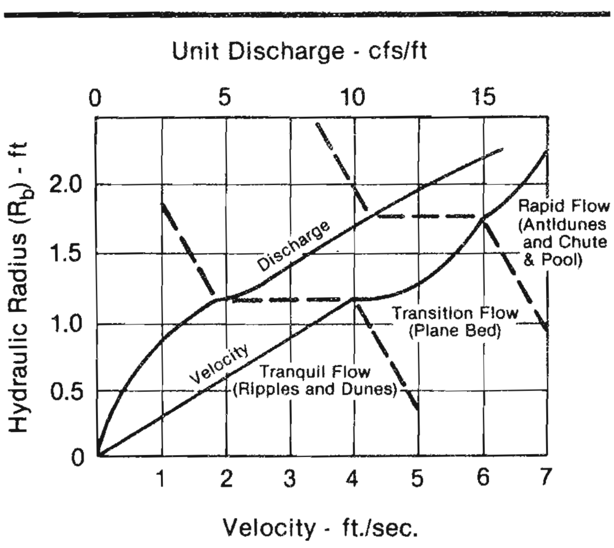

Figure 4–17.—Depth-discharge relationships obtained by Alam-Kennedy technique.